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FINITE ELEMENT FORMULATION AND SOLUTION OF CONTACT-IMPACT PROBLEMS IN CONTINUUM MECHANICS

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FINITE ELEMENT FORMULATION AND SOLUTION OF CONTACT-IMPACT PROBLEMS IN CONTINUUM MECHANICS

May 1974

An Investigation Conducted by

STRUCTURAL ENGINEERING LABORATORY
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This report presents a general theory of contact-impact problems cast in a variational theorem suitable for implementation with the finite element method. The numerical scheme is described as is the structural analysis computer code in which it is contained.

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Introduction

In this report we consider the general problem of contact and impact between two bodies. The report is divided into three basic parts. These parts describe: (I) The general theory of contact-impact problems, (II) A numerical scheme for the analysis of contact-impact problems, and (III) The description of computer program FEAP 74 for the solution of contact-impact problems. In an appendix we include the program subroutines and general input description for FEAP 74.

In Sections 1 to 6, Part I, we deal with spatial aspects of the theory and in Section 7, Part I, we deal with temporal aspects. This splitting of the theory is motivated by the way we intend to numerically solve the equations, i.e., the finite element method spatially and a finite difference method temporally.

Part II considers a numerical implementation of the theory given in Part I. Section 9 deals with spatial notions of the numerical problem and Section 10 the temporal. The solution scheme for the resulting algebraic problem is discussed in Sections 11 and 12.

The computer program FEAP 74 was modified to incorporate the numerical contact-impact model. The program modifications and capabilities together with two numerical examples are contained in Part III.

Finally, in the appendix we give listings for the contact subroutines together with the data input instructions.

PART I

VARIATIONAL FORMULATION OF CONTACT-IMPACT PROBLEMS IN CONTINUUM MECHANICS

1. Preliminaries

Our conventions on indices are as follows:

Superscripts indicate to which body an entity pertains. Summation is to take place only when explicitly indicated.

Latin subscripts range over 1,2,3, while Greek subscripts range over 1,2. The summation convention is assumed to hold for both.

A body $\mathcal{B}$ is a nice connected region of $\mathbb{R}^3$ with a piecewise smooth boundary $\partial\mathcal{B}$. A contact* problem is a boundary value problem, or an initial-boundary value problem, in which two bodies, $\mathcal{B}^1$ and $\mathcal{B}^2$, interact according to the principles of mechanics. Thus the primary kinematic axiom of a contact problem is that configurations $\mathcal{b}^1$ and $\mathcal{b}^2$, of $\mathcal{B}^1$ and $\mathcal{B}^2$, respectively, do not penetrate each other, i.e.,

$$
\begin{aligned}
(\mathcal{b}^1)^\circ \cap \mathcal{b}^2 &= \emptyset, \\
\mathcal{b}^1 \cap (\mathcal{b}^2)^\circ &= \emptyset,
\end{aligned}
\tag{1}
$$

where $(\mathcal{b}^\alpha)^\circ$ denotes the interior of $\mathcal{b}^\alpha$, $\alpha = 1,2$.

On the other hand the unique conditon which characterizes contact problems is that material points on the boundaries of $\mathcal{B}^1$ and $\mathcal{B}^2$ may coalesce during the motion of the bodies. Thus we say $\mathcal{B}^1$ and $\mathcal{B}^2$ are in contact if $\partial\mathcal{b}^1 \cap \partial\mathcal{b}^2 \neq \emptyset$, and we define the contact surface $\mathcal{e}$ by

---

* It is usual for the term contact to have a static connotation while the term impact has a dynamic connotation. We shall use contact in the general sense to include static as well as dynamic phenomena.

$$\mathscr{C} = \partial \mathscr{b}^1 \cap \partial \mathscr{b}^2 \quad . \tag{2}$$

If $\mathscr{B}^1$ and $\mathscr{B}^2$ are never in contact then $\mathscr{C} = \emptyset$ for all configurations $\mathscr{b}^1$ and $\mathscr{b}^2$, and in this case an initial-boundary value problem for $\mathscr{B}^1$ and $\mathscr{B}^2$ reduces to one in which $\mathscr{B}^1$ and $\mathscr{B}^2$ may be treated separately. Thus a non-trivial contact problem is one in which $\mathscr{C} \neq \emptyset$ for at least one instant during the motion of $\mathscr{B}^1$ and $\mathscr{B}^2$. The picture (Fig. 1) illustrates these notions.

Equation (1) implies that $\mathscr{C}$ is a material surface with respect to both bodies, i.e., one which is not crossed by material particles. From this we may deduce the interface conditions on $\mathscr{C}$.

Let $\underset{\sim}{x}$ be a <u>persistent point</u> of $\mathscr{C}$ (one at which joining or releasing of the bodies is not instantaneously occurring) and $\underset{\sim}{v}$ be the velocity of $\underset{\sim}{x}$ ($\underset{\sim}{v} = \dot{\underset{\sim}{x}}$). Note that only the normal part of $\underset{\sim}{v}$ is independent of the parametrization of $\mathscr{C}$. Let $\underset{\sim}{v}^1$ and $\underset{\sim}{v}^2$ be the velocities of the material particles located at the points $\underset{\sim}{x}^1$ and $\underset{\sim}{x}^2$, contained in $\partial \mathscr{b}^1$ and $\partial \mathscr{b}^2$, respectively, such that $\underset{\sim}{x} = \underset{\sim}{x}^1 = \underset{\sim}{x}^2$ at the present instant. Then since $\mathscr{C}$ is material and $\underset{\sim}{x}$ is persistant

$$\underset{\sim}{v} \cdot \underset{\sim}{n} = \underset{\sim}{v}^1 \cdot \underset{\sim}{n} = \underset{\sim}{v}^2 \cdot \underset{\sim}{n} \quad , \tag{3}$$

where $\underset{\sim}{n}$ is a unit normal vector to $\mathscr{C}$ at $\underset{\sim}{x}$. From this it follows that a necessary condition for momentum to be balanced at $\underset{\sim}{x}$ is that

$$(\underset{\sim}{t}^1 + \underset{\sim}{t}^2) \cdot \underset{\sim}{n} = \underset{\sim}{0} \quad , \tag{4}$$

where $\underset{\sim}{t}^\alpha$ is the Cauchy traction vector with respect to $\partial \mathscr{b}^\alpha$.

In addition we assume that no tensile tractions can occur on $\mathscr{C}$,

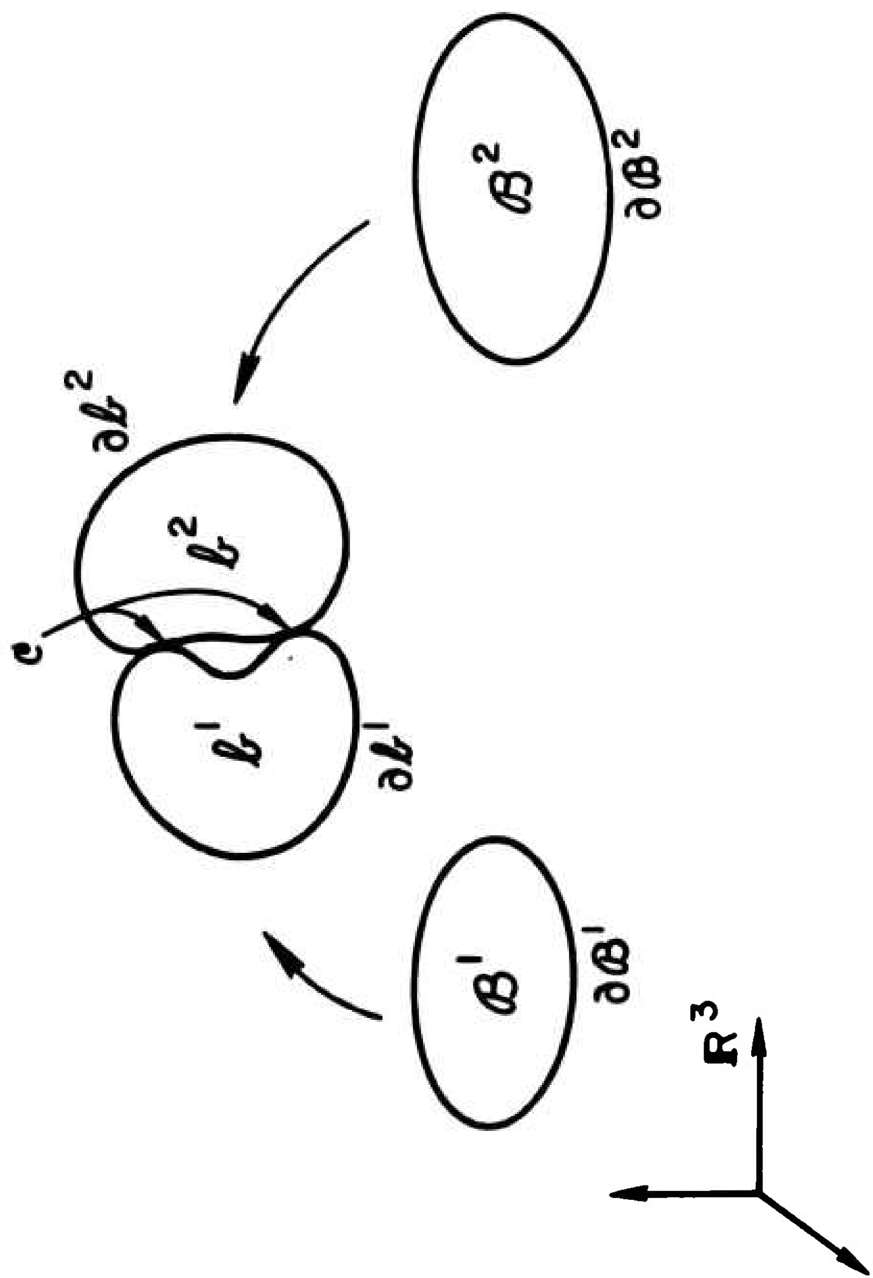

$\mathcal{B}^2$
$\partial\mathcal{B}^2$
∂b^2
b^2
c
b^1
∂b^1
$\mathcal{B}^1$
$\partial\mathcal{B}^1$
$\mathbf{R}^3$

Figure 1

$$\underset{\sim}{t}^{\alpha} \cdot \underset{\sim}{n}^{\alpha} \leq 0 \ , \tag{5}$$

where $\underset{\sim}{n}^{\alpha}$ is the outward unit normal vector to $\partial \mathcal{B}^{\alpha}$. This condition excludes the possibility of the two bodies being glued together. Conditions (1-5) characterize our notion of a contact problem.

Note that thus far we have said nothing about the tangential parts of $\underset{\sim}{v}^{\alpha}$ and $\underset{\sim}{t}^{\alpha}$. These remaining conditions are determined by the frictional nature of the contact. We shall study two simple cases.

<u>Case I</u>: If we assume that points, once in contact, move with $\mathfrak{C}$ until released, we have that

$$\underset{\sim}{v}^{1} = \underset{\sim}{v}^{2} \ , \tag{6}$$

and therefore

$$\underset{\sim}{t}^{1} + \underset{\sim}{t}^{2} = \underset{\sim}{0} \ . \tag{7}$$

For this model we say that a no-slip, or perfect friction, condition is achieved on $\mathfrak{C}$. Thus condition (5) and equations (6) and (7) are the interface conditions for this case.

<u>Case II</u>: We may create the interface conditions for a frictionless, sliding contact by asserting that the tangential part of each $\underset{\sim}{t}^{\alpha}$ is identically zero,

$$\underset{\sim}{t}^{\alpha} - (\underset{\sim}{t}^{\alpha} \cdot \underset{\sim}{n}^{\alpha})\underset{\sim}{n}^{\alpha} = \underset{\sim}{0} \ . \tag{8}$$

Eq. (8), along with (3-5), are the interface conditions for this case.

2. Variational Theorems

We will formulate a variational theorem for the contact problem of finite elastodynamics. We point out, however, that our treatment is entirely general and could be used in conjunction with any field theory, as the only unique feature of the formulation involves the handling of interface conditions. At the same time finite elastodynamics, though lending itself to a clean and simple variational statement, is a case of wide practical interest.

We shall first obtain a variational theorem for the usual initial-boundary value problem of finite elastodynamics by a trivial generalization of some work done by S. Nemat-Nasser [1].

For notational simplicity let $\mathcal{A}$ denote $\partial\mathcal{B}$, and let $d\mathcal{A}$ and $d\mathcal{B}$ denote area and volume forms for $\mathcal{B}$ and $\mathcal{A}$, respectively. Let $\mathcal{A}_T \subset \mathcal{A}$ be that part of $\mathcal{A}$ where surface tractions are prescribed, and denote by $\bar{\underset{\sim}{T}}$ the Piola - Kirchhoff traction vector representing these prescribed tractions. Call ρ_0 the density of $\mathcal{B}$ in the initial configuration, $\underset{\sim}{F}$ the extrinsic body force vector and let $\underset{\sim}{x} = \underset{\sim}{x}_t(\underset{\sim}{X})$ represent the position at time t of the material particle located at $\underset{\sim}{X}$ in the initial configuration. For convenience we take $\mathcal{B}$ to be the initial configuration. We denote by $\partial\underset{\sim}{x}/\partial\underset{\sim}{X}$ the deformation gradients and by $\Phi(\partial\underset{\sim}{x}/\partial\underset{\sim}{X})$ the strain energy density. Then if $\underset{\sim}{x}$ satisfies the kinematic boundary conditions

$$\underset{\sim}{x} = \bar{\underset{\sim}{x}} \tag{9}$$

on $\mathcal{A}_x \subset \mathcal{A}$, where

$$\mathcal{A}_x \cup \mathcal{A}_T = \mathcal{A}\,,$$
$$\mathcal{A}_x \cap \mathcal{A}_T = \varnothing\,,$$

the functional Π defined by

$$\Pi(\underset{\sim}{x}) = \int_0^t \Big\{ \int_{\mathcal{B}} \big(\Phi(\partial \underset{\sim}{x}/\partial \underset{\sim}{X}) - \rho_0 \dot{\underset{\sim}{x}} \cdot \dot{\underset{\sim}{x}}/2 - \rho_0 \underset{\sim}{F} \cdot \underset{\sim}{x} \big) d\mathcal{B} - \int_{\mathcal{A}_T} \underset{\sim}{x} \cdot \bar{\underset{\sim}{T}} \, d\mathcal{A} \Big\} dt \, , \tag{10}$$

is stationary, i.e., its first variation vanishes

$$0 = \delta\Pi(\underset{\sim}{x}, \delta\underset{\sim}{x}) = \int_0^t \Big\{ \int_{\mathcal{B}} \big(\rho_0 (\ddot{\underset{\sim}{x}} - \underset{\sim}{F}) - \mathrm{DIV}\, \underset{\sim}{P} \big) \cdot \delta\underset{\sim}{x} \, d\mathcal{B} + \int_{\mathcal{A}_T} (\underset{\sim}{T} - \bar{\underset{\sim}{T}}) \cdot \delta\underset{\sim}{x} \, d\mathcal{A} \Big\} dt \, , \tag{11}$$

subject to the constraint on variations $\delta\underset{\sim}{x}_t = \delta\underset{\sim}{x}_0 = \underset{\sim}{0}$, (12)

if and only if the equations of motion and traction boundary conditions are satisfied

$$\rho_0 (\ddot{\underset{\sim}{x}} - \underset{\sim}{F}) = \mathrm{DIV}\, \underset{\sim}{P} \, , \qquad \text{in } \mathcal{B} \, , \tag{13}$$

$$\underset{\sim}{T} = \bar{\underset{\sim}{T}} \, , \qquad \text{on } \mathcal{A}_T \, , \tag{14}$$

where $\underset{\sim}{P} = \partial\Phi/\partial(\partial\underset{\sim}{x}/\partial\underset{\sim}{X})$ is the first Piola - Kirchhoff stress tensor, $\underset{\sim}{T} = \underset{\sim}{N} \cdot \underset{\sim}{P}$ is the Piola - Kirchhoff traction vector, and $\underset{\sim}{N}$ is the outward unit normal vector to $\mathcal{A}$. The solution to the initial-boundary value problem must also satisfy the given initial conditions

$$\left. \begin{aligned} \underset{\sim}{x} &= \underset{\sim}{x}_0 \\ \underset{\sim}{v} &= \underset{\sim}{v}_0 \end{aligned} \right\} \quad \text{in } \mathcal{B} \, . \tag{15}$$

To interpret this variational theorem for two (non-interacting) bodies set

$$\mathcal{B} = \mathcal{B}^1 \cup \mathcal{B}^2 \, ,$$

$$\mathcal{A} = \mathcal{A}^1 \cup \mathcal{A}^2 \, , \quad \text{etc.} \, ,$$

and write

$$\Pi(\underset{\sim}{x}) = \Pi^1(\underset{\sim}{x}^1) + \Pi^2(\underset{\sim}{x}^2) .$$

The next step is to add to Π terms manifesting the interface conditions on $\mathcal{C}$ and to stipulate the constraints under which the vanishing of the first variation of the appended functional corresponds to a solution of the contact problem. To do this we must consider further the kinematics and geometry of $\mathcal{C}$.

Define two piecewise smooth, invertible maps $\underset{\sim}{\chi}^1, \underset{\sim}{\chi}^2$ by the condition

$$(\underset{\sim}{\chi}^\alpha)^{-1} : \mathcal{C} \longrightarrow \mathcal{C}^\alpha \subset \mathcal{A}^\alpha , \tag{16}$$

where each $\underset{\sim}{\chi}^\alpha$ identifies points on the boundary of the initial configuration $\mathcal{B}^\alpha$ which map into the contact surface $\mathcal{C}$ at each instant of time. If $\underset{\sim}{x} \in \mathcal{C}$, then $\underset{\sim}{X}^1 = (\underset{\sim}{\chi}^1)^{-1}(\underset{\sim}{x})$ and $\underset{\sim}{X}^2 = (\underset{\sim}{\chi}^2)^{-1}(\underset{\sim}{x})$ are the positions of particles in $\mathcal{A}^1$ and $\mathcal{A}^2$, respectively, which have coalesced at $\underset{\sim}{x} \in \mathcal{C}$. It is clear what the $\underset{\sim}{\chi}^\alpha$'s really are, viz., if $\underset{\sim}{x}^\alpha = \underset{\sim}{x}_t^\alpha(\underset{\sim}{X}^\alpha)$, for all $\underset{\sim}{X}^\alpha \in \mathcal{B}^\alpha$, represents the motion of body $\mathcal{B}^\alpha$ from the original configuration $\mathcal{B}^\alpha$ to the present one $\mathcal{A}^\alpha$, then $\underset{\sim}{\chi}^\alpha$ is the restriction of $\underset{\sim}{x}^\alpha$ to $\mathcal{C}^\alpha$,

$$\underset{\sim}{\chi}^\alpha(\underset{\sim}{X}^\alpha) = \underset{\sim}{x}^\alpha(\underset{\sim}{X}^\alpha) \quad , \tag{17}$$

for each $\underset{\sim}{X}^\alpha \in \mathcal{C}^\alpha$, $\alpha = 1,2$. For the time being we consider the $\underset{\sim}{\chi}^\alpha$'s as maps defined independently of the $\underset{\sim}{x}^\alpha$'s and consider (17) a constraint on possible motions.

We are interested in to what extent the relation

$$\underset{\sim}{x} = \underset{\sim}{\chi}^1(\underset{\sim}{X}^1) = \underset{\sim}{\chi}^2(\underset{\sim}{X}^2) \quad , \tag{18}$$

is smooth in time and analogously under what circumstances the variations of the $\underset{\sim}{\chi}^\alpha$'s are equal. In general the $\underset{\sim}{\chi}^\alpha$'s will not even be continuous in time since contact surfaces can be instantaneously created or destroyed. If we eliminate such exceptional instants and consider only persistent points, the bodies still may slide with respect to each other, as depicted in Fig. 2. Thus tangential velocities are seen to be unequal in general. However, when $\underset{\sim}{x}$ is persistent, the impenetrability condition (1) forces the normal velocity components to be equal, and concomitantly the normal components of variations of the $\underset{\sim}{\chi}^\alpha$'s are also equal

$$\delta\underset{\sim}{\chi}^1 \cdot \underset{\sim}{n} = \delta\underset{\sim}{\chi}^2 \cdot \underset{\sim}{n} \tag{19}$$

For sliding contact (Case II), Eq. (19) characterizes the constraint on variations of the $\underset{\sim}{\chi}^\alpha$'s eqivalent to the velocity constraint (3).

For no-slip contact (Case I),

$$\delta\underset{\sim}{\chi}^1 = \delta\underset{\sim}{\chi}^2 \quad , \tag{20}$$

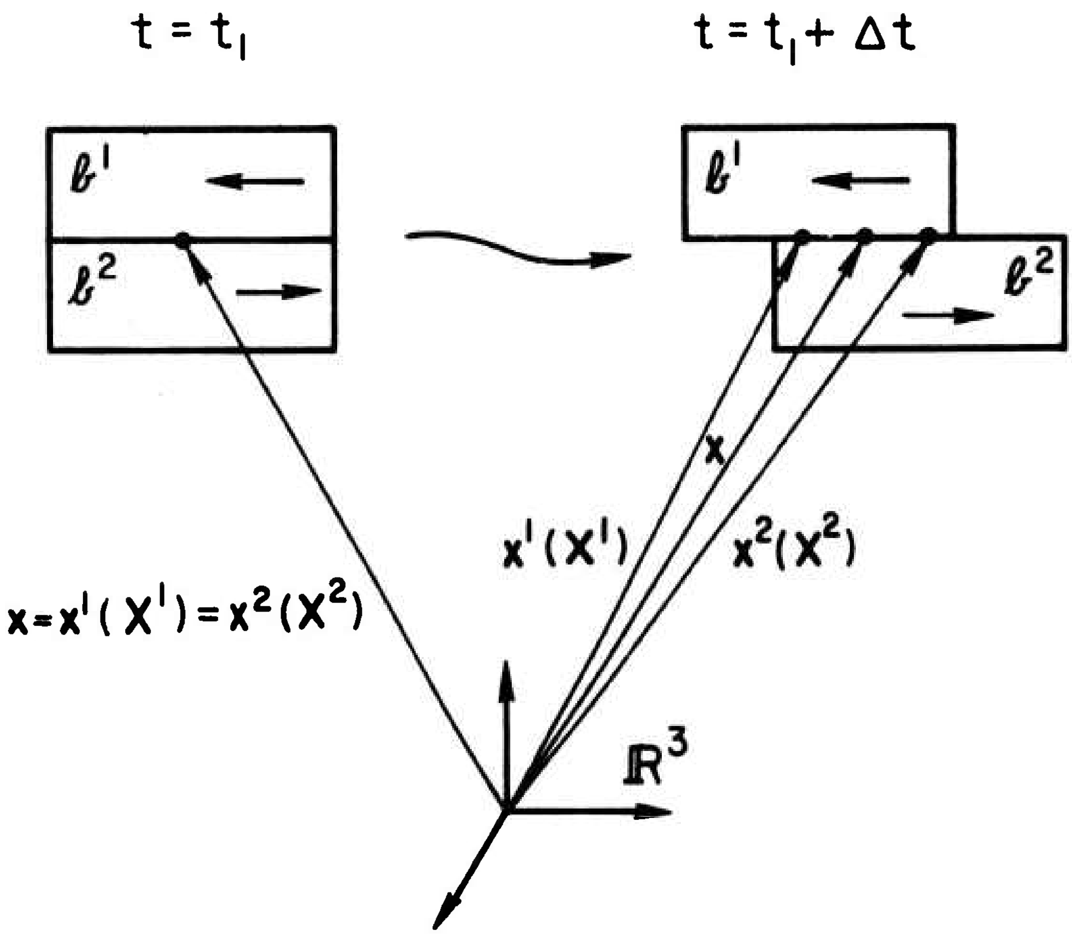
is easily seen to be the condition on variations equivalent to Eq. (6). We shall see that Eqs. (19) and (20) lead to the proper interface conditions in the variational theorems.

Introduce vector valued Lagrange multipliers $\underset{\sim}{\tau}^\alpha$, and add

$$\mathcal{K} = -\sum_{\alpha=1}^{2} \int_0^t \int_{\mathcal{C}^\alpha} \underset{\sim}{\tau}^\alpha \cdot (\underset{\sim}{x}^\alpha - \underset{\sim}{\chi}^\alpha)\, d\mathcal{C}^\alpha dt \quad , \tag{21}$$

to the functional Π (Eq. 10). Note that when $\mathcal{C} \neq \emptyset$,

$$\mathcal{A}^\alpha = \mathcal{A}^\alpha_x \cup \mathcal{A}^\alpha_T \cup \mathcal{C}^\alpha \quad ,$$



$t = t_1$
$t = t_1 + \Delta t$
$\mathcal{B}^1$
$\mathcal{B}^2$
$\mathcal{B}^1$
$\mathcal{B}^2$
x
$x^1(X^1)$
$x^2(X^2)$
$x = x^1(X^1) = x^2(X^2)$
$\mathbb{R}^3$

Figure 2

and assume for consistency's sake that

$$\mathcal{C}^{\alpha} \subset \mathcal{A}_T^{\alpha} ,$$

$$\underset{\sim}{T} = \underset{\sim}{0} \qquad \text{on} \quad \mathcal{C}^{\alpha}. \tag{22}$$

This condition will preclude the ambiguous circumstance of non-zero tractions being specified on the contact area. Upon taking variations of $\mathcal{J} = \Pi + \mathcal{K}$ we get Eqs. (13), (14) and,

$$\begin{aligned} 0 = & -\sum_{\alpha=1}^{2} \int_0^t \int_{\mathcal{C}^{\alpha}} \Big\{ \delta\underset{\sim}{\tau}^{\alpha} \cdot (\underset{\sim}{x}^{\alpha} - \underset{\sim}{\chi}^{\alpha}) + \\ & + \delta\underset{\sim}{x}^{\alpha} \cdot (\underset{\sim}{\tau}^{\alpha} - \underset{\sim}{T}^{\alpha}) - \delta\underset{\sim}{\chi}^{\alpha} \cdot \underset{\sim}{\tau}^{\alpha} \Big\} d\mathcal{C}^{\alpha} dt \quad + \\ & + \text{transversality condition.} \end{aligned} \tag{23}$$

The transversality condition is the classical terminology for variations associated with the domain $\mathcal{C}^{\alpha}$.

The first summand of (23) gives us (17) which insures that the $\underset{\sim}{x}^{\alpha}$'s map into $\mathcal{C}$ properly. The second summand identifies $\underset{\sim}{\tau}^{\alpha}$ as the Piola - Kirchhoff traction vector $\underset{\sim}{T}^{\alpha}$ on $\mathcal{C}^{\alpha}$. Let us investigate the third summand.

Consider first Case I and define

$$\delta\underset{\sim}{\chi} = \delta\underset{\sim}{\chi}^{\alpha} \quad , \quad \alpha = 1,2 , \tag{24}$$

which makes sense because of Eq. (20). This condition is equivalent to insisting

$$\dot{\underset{\sim}{\chi}} = \dot{\underset{\sim}{\chi}}^{\alpha} \quad , \quad \alpha = 1,2 ,$$

thus the first summand of (23) also implies (6) holds whenever we have a

persistent point. Let j^{α} denote the Jacobian determinant associated with χ^{α},

$$da = j^{\alpha} d\mathcal{C}^{\alpha}. \tag{25}$$

Notice then that since $\underset{\sim}{\tau}^{\alpha}$ is the Piola - Kirchhoff traction vector, $(1/j^{\alpha})\underset{\sim}{\tau}^{\alpha}$ is the corresponding Cauchy traction vector. With these we have for the third summand,

$$0 = \sum_{\alpha=1}^{2} \int_{\mathcal{C}^{\alpha}} \delta\underset{\sim}{\chi}^{\alpha} \cdot \underset{\sim}{\tau}^{\alpha} d\mathcal{C}^{\alpha} = \int_{a} \delta\underset{\sim}{\chi} \cdot ((1/j^{1})\underset{\sim}{\tau}^{1} + (1/j^{2})\underset{\sim}{\tau}^{2})da \quad , \tag{26}$$

which in words means the Cauchy traction vectors are in equilibrium. Thus the momentum balance, Eq. (7), is satisfied on a.

In Case II we only have that (19) holds, so define

$$\delta\chi(n) = \delta\underset{\sim}{\chi}^{\alpha} \cdot \underset{\sim}{n} \quad , \quad \alpha = 1,2 . \tag{27}$$

This requirement also insures that,

$$\dot{\underset{\sim}{\chi}}^{1} \cdot \underset{\sim}{n} = \dot{\underset{\sim}{\chi}}^{2} \cdot \underset{\sim}{n} \quad ,$$

thus the first summand of (23) implies (3). For this case the third summand takes the form,

$$\begin{aligned} 0 = \sum_{\alpha=1}^{2} \int_{\mathcal{C}^{\alpha}} \delta\underset{\sim}{\chi}^{\alpha} \cdot \underset{\sim}{\tau}^{\alpha} d\mathcal{C}^{\alpha} &= \int_{a} \delta\chi(n)((1/j^{1})\underset{\sim}{\tau}^{1} \cdot \underset{\sim}{n} + (1/j^{2})\underset{\sim}{\tau}^{2} \cdot \underset{\sim}{n})da \\ &+ \sum_{\alpha=1}^{2} \int_{\mathcal{C}^{\alpha}} (\delta\underset{\sim}{\chi}^{\alpha} - \delta\chi(n)\underset{\sim}{n}) \cdot \underset{\sim}{\tau}^{\alpha} d\mathcal{C}^{\alpha}. \end{aligned} \tag{28}$$

The integral over a gives us Eq. (4). The significance of the second integral hinges on the observation that $(\delta\underset{\sim}{\chi}^{\alpha} - \delta\chi(n)\underset{\sim}{n})$ is a tangent vector to a for each α. Thus the tangential part of each $\underset{\sim}{\tau}^{\alpha}$ is identically zero, which is equivalent to the shear free condition, Eq. (8),

which we require for Case II.

A standard calculation enables us to write the transversality condition as,

$$0 = \sum_{\alpha=1}^{2} \int_{\partial\mathcal{C}^\alpha} (\delta\boldsymbol{\chi}^\alpha \cdot \mathbf{T}^\alpha)(\boldsymbol{\tau}^\alpha \cdot (\mathbf{x}^\alpha - \boldsymbol{\chi}^\alpha))\, d(\partial\mathcal{C}^\alpha) \quad , \tag{29}$$

where the transversal $\mathbf{T}^\alpha$ is a unit vector field tangent to $\mathcal{C}^\alpha$, and perpendicular and pointing outward with respect to $\partial\mathcal{C}^\alpha$, Fig. 3. Thus (29) implies that

$$\boldsymbol{\tau}^\alpha \cdot (\mathbf{x}^\alpha - \boldsymbol{\chi}^\alpha) = 0 \qquad \text{on } \partial\mathcal{C}^\alpha, \ \alpha = 1,2 \tag{30}$$

Assuming continuity of the integrands of (21) on the closure of $\mathcal{C}^\alpha$, condition (30) is already implied by the first summand of (23). This assumption precludes $\boldsymbol{\tau}^\alpha$ taking the form of a δ-distribution on $\partial\mathcal{C}^\alpha$.

Although this assumption is warranted here it may not be true when one employs certain approximate theories in mechanics. For instance consider the case where a Bernoulli-Euler beam is uniformly loaded and sits on a rigid parabolic surface (Fig. 4). At the contact points a, a', concentrated reactions must exist to balance shear forces. This example is actually from a completely different class of contact problems in that contact is made along a part of the interior rather than the boundary. Such problems as the contact of plates and shells also fall into this class. We could summarize such situations by the description -- m-dimensional contact of m-dimensional bodies, e.g., for the beam m=1, and for plates and shells m=2. The case under investigation in this paper (m=3) is an example of the (m-1)-dimensional contact of m-dimensional bodies.

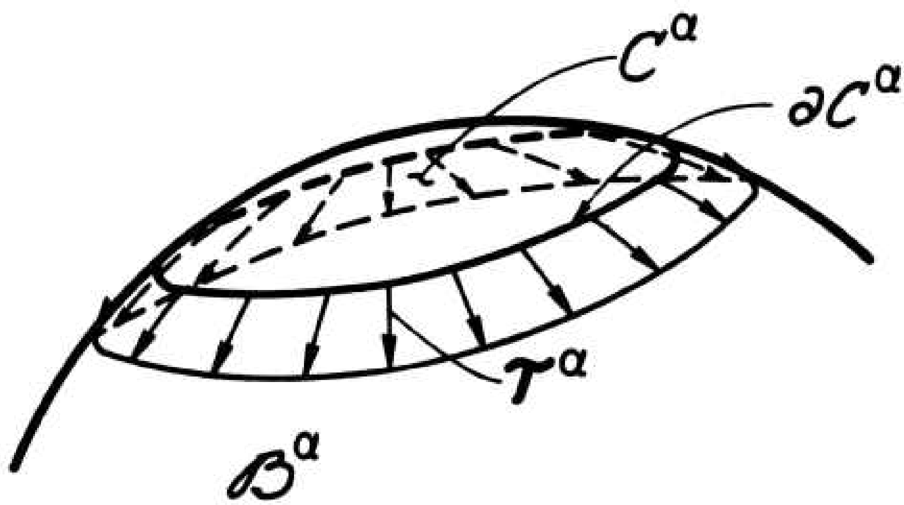


Figure 3

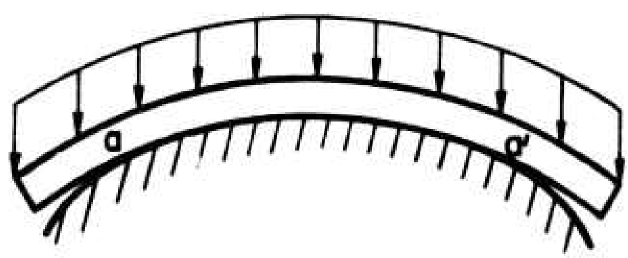


Figure 4

It is good to keep in mind cases such as that illustrated in Fig. 4 when considering specific boundary value problems.

A further point worth mentioning here is that the transversality condition will in general be an independent one in a numerical algorithm. For example, if the fields in the integrand of (21) are approximated by a family of trial functions, Eq. (23) only implies that some weighted integrals over the $\mathcal{C}^{\alpha}$'s vanish. The condition (29) requires that weighted integrals over the $\partial\mathcal{C}^{\alpha}$'s also vanish.

We now summarize our results in the following theorems:

Theorem I: Let (1), (2), (5), (9), (12), (15), and (20) hold. Then $\underset{\sim}{x}$ is a solution to the no-slip contact problem (Case I), that is, (6), (7), (13), (14) and (17) also hold if, and only if, $\delta\mathcal{J}=0$ for arbitrary variations of $\underset{\sim}{x}^{\alpha}$, $\underset{\sim}{\chi}^{\alpha}$ and $\underset{\sim}{\tau}^{\alpha}$, $\alpha=1,2$.

Theorem II: Let (1), (2), (5), (9), (12), (15), and (19) hold. Then $\underset{\sim}{x}$ is a solution to the sliding contact problem (Case II), that is, (3), (4), (8), (13), (14) and (17) also hold if, and only if, $\delta\mathcal{J}=0$ for arbitrary variations of $\underset{\sim}{x}^{\alpha}$, $\underset{\sim}{\chi}^{\alpha}$ and $\underset{\sim}{\tau}^{\alpha}$, $\alpha=1,2$.

3. Consideration of Theorems I and II as Computational Tools

Theorems I and II may be employed to generate numerical algorithms for the solution of contact problems. The basic idea is to represent $\underset{\sim}{x}^{\alpha}$, $\underset{\sim}{\lambda}^{\alpha}$ and $\underset{\sim}{\zeta}^{\alpha}$ as the product of known functions on $\mathbb{R}^3$ with unknown parameters depending on time. Then Theorems I and II provide us with a method for generating an approximate system of equations (e.g., by the classical Ritz-Galerkin technique) in terms of these unknown parameters, which then can be solved incrementally and/or iteratively, subject to the side conditions of the theorems. The constraints (1) and (5) will both take the form of inequalities in actual computations, thus the ideas of optimization theory will probably be useful in the actual construction of a numerical algorithm.

The finite element method is a powerful technique for obtaining a system of approximate equations, and it is of interest to find out how amenable are Theorems I and II to a finite element formulation. Unfortunately the term $\mathcal{X}$ would result in a terrible mess if the integrand was represented by typical finite element functions. This is because the boundaries of the $\mathcal{C}^{\alpha}$'s are unknown and thus a parametric integration would bury the defining parameters of the $\mathcal{C}^{\alpha}$'s in the arguments of Heaviside functions representing the supports of the elements. Note that a classical Ritz-Galerkin approximation would not be subject to this pitfall, since the associated trial functions could be chosen to be real analytic and thus easily integrated parametrically to a relatively simple form. However, such a formulation is restricted to a geometrically simpler class of problems. Thus it is desirable to seek a generalization that will lend itself cleanly to a finite element formulation.

4. Variational Theorems Without Transversality Conditions

Let $\tilde{\mathcal{C}}^\alpha$ be a fixed part of $\mathcal{Q}_\tau^\alpha$ such that

$$\tilde{\mathcal{C}}^\alpha \supset \mathcal{C}^\alpha \quad , \tag{31}$$

and

$$\underset{\sim}{T} = \underset{\sim}{0} \quad \text{on} \quad \tilde{\mathcal{C}}^\alpha \sim \mathcal{C}^\alpha . \tag{32}$$

Define a scalar valued function η^α on $\tilde{\mathcal{C}}^\alpha$ such that

$$\eta^\alpha(\underset{\sim}{X}^\alpha) = 0 \qquad \text{if} \quad \underset{\sim}{X}^\alpha \in \tilde{\mathcal{C}}^\alpha \sim \mathcal{C}^\alpha . \tag{33}$$

Let $\tilde{\varepsilon} \supset \varepsilon$, and define the maps $\underset{\sim}{\chi}^\alpha$ by the condition

$$(\underset{\sim}{\chi}^\alpha)^{-1} : \tilde{\varepsilon} \longrightarrow \tilde{\mathcal{C}}^\alpha ,$$

where, as before, $\underset{\sim}{\chi}^\alpha$ represents $\underset{\sim}{x}^\alpha$ on $\mathcal{C}^\alpha$; but on $\tilde{\mathcal{C}}^\alpha \sim \mathcal{C}^\alpha$ we place no physical interpretation on $\underset{\sim}{\chi}^\alpha$. Thus on $\tilde{\mathcal{C}}^\alpha$ we will always have that,

$$\eta^\alpha(\underset{\sim}{x}^\alpha - \underset{\sim}{\chi}^\alpha) = \underset{\sim}{0} \quad , \tag{34}$$

since $\underset{\sim}{x}^\alpha = \underset{\sim}{\chi}^\alpha$ on $\mathcal{C}^\alpha$ and $\eta^\alpha = 0$ on the relative complement $\tilde{\mathcal{C}}^\alpha \sim \mathcal{C}^\alpha$.

Introduce vector valued Lagrange multipliers $\underset{\sim}{\sigma}^\alpha$ and let $\mathcal{L} = \mathrm{II} + \mathcal{M}$ where

$$\mathcal{M} = - \sum_{\alpha=1}^{2} \int_0^t \int_{\tilde{\mathcal{C}}^\alpha} \underset{\sim}{\sigma}^\alpha \cdot \eta^\alpha (\underset{\sim}{x}^\alpha - \underset{\sim}{\chi}^\alpha) \, d\tilde{\mathcal{C}}^\alpha dt . \tag{35}$$

We require that the variations of $\underset{\sim}{\chi}^\alpha$ satisfy the same conditions as before, but now for all $\tilde{\mathcal{C}}^\alpha$:

$$\left.\begin{array}{ll} \text{Case I:} & \delta\underset{\sim}{\chi} \overset{\text{def.}}{=} \delta\underset{\sim}{\chi}^\alpha \\ \text{Case II:} & \delta\chi(n) \overset{\text{def.}}{=} \delta\underset{\sim}{\chi}^\alpha \cdot \underset{\sim}{n} \end{array}\right\} \quad \text{on} \quad \tilde{\mathcal{C}}^\alpha \tag{36}$$

where $\underset{\sim}{n}$ is a unit normal vector to $\tilde{\mathcal{E}}$. Computing the first variation of $\mathcal{L}$ we have the usual conditions emanating from II and

$$
\begin{aligned}
0 = -\sum_{\alpha=1}^{2} \int_0^t \int_{\tilde{\mathcal{E}}^\alpha} \Big\{ & \delta \underset{\sim}{\sigma}^\alpha \cdot (\eta^\alpha (\underset{\sim}{x}^\alpha - \underset{\sim}{\chi}^\alpha)) \\
& + \delta \eta^\alpha (\underset{\sim}{\sigma}^\alpha \cdot (\underset{\sim}{x}^\alpha - \underset{\sim}{\chi}^\alpha)) \\
& + \delta \underset{\sim}{x}^\alpha \cdot (\eta^\alpha \underset{\sim}{\sigma}^\alpha - \underset{\sim}{T}^\alpha) \\
& - \delta \underset{\sim}{\chi}^\alpha \cdot (\eta^\alpha \underset{\sim}{\sigma}^\alpha) \Big\} \, d\tilde{\mathcal{E}}^\alpha dt .
\end{aligned}
\tag{37}
$$

The first summand gives us (34) and we define

$$
\mathcal{C}^\alpha = \{ \underset{\sim}{X}^\alpha \in \tilde{\mathcal{E}}^\alpha : \underset{\sim}{x}^\alpha(\underset{\sim}{X}^\alpha) = \underset{\sim}{\chi}^\alpha(\underset{\sim}{X}^\alpha) \} . \tag{38}
$$

The third summand defines $\eta^\alpha \underset{\sim}{\sigma}^\alpha$ as the Piola - Kirchhoff traction vector. Note that this insures that $\underset{\sim}{T} = \underset{\sim}{0}$ on $\tilde{\mathcal{E}}^\alpha \sim \mathcal{C}^\alpha$ since $\eta^\alpha = 0$ there. The fourth summand gives us the appropriate Cauchy traction condition across $\mathcal{C}$ for each case of (36). The second summand is identically satisfied on $\mathcal{C}^\alpha$ since $\underset{\sim}{x}^\alpha = \underset{\sim}{\chi}^\alpha$. On $\tilde{\mathcal{E}}^\alpha \sim \mathcal{C}^\alpha$ it tells us that $\underset{\sim}{\sigma}^\alpha$ is orthogonal to $\underset{\sim}{x}^\alpha - \underset{\sim}{\chi}^\alpha$, but this is of no physical interest.

Thus we can state the following theorems:

<u>Theorem I'</u>: Let (1), (2), (5), (9), (12), (15), (31), (32) and $(36)_1$ hold. Then $\underset{\sim}{x}$ is a solution to the no-slip contact problem (Case I), that is, (6), (7), (13), (14) and (17) also hold where $\mathcal{C}^\alpha$ is defined by (38), if $\delta\mathcal{L} = 0$ for arbitrary variations of $\underset{\sim}{x}^\alpha$, $\underset{\sim}{\chi}^\alpha$, η^α and $\underset{\sim}{\sigma}^\alpha$, $\alpha = 1, 2$.

<u>Theorem II'</u>: Let (1), (2), (5), (9), (12), (15), (31), (32) and $(36)_2$ hold. Then $\underset{\sim}{x}$ is a solution to the sliding contact problem (Case II), that is, (3), (4), (8), (13), (14) and (17) hold where $\mathcal{C}^\alpha$ is defined by (38), if $\delta\mathcal{L} = 0$ for arbitrary variations of $\underset{\sim}{x}^\alpha$, $\underset{\sim}{\chi}^\alpha$, η^α and $\underset{\sim}{\sigma}^\alpha$, $\alpha = 1, 2$.

The important feature of these theorems is that the regions $\tilde{\mathcal{E}}^{\alpha}$ are fixed. Thus transversality conditions are absent, and the theorems may be applied to finite element formulations. In fact one would naturally take $\tilde{\mathcal{E}}^{\alpha}$ to be a union of elements in $\mathcal{A}^{\alpha}$, large enough to contain $\mathcal{C}^{\alpha}$ throughout the motion.

Thus far our considerations have been quite general and, in fact, more general than would be required for the solution of particular classes of contact problems. In the next section we illustrate the many simplifications which can be made in the application of the preceeding theorems to a class of problems of wide practical interest.

5. Hertzian Contact Problems

We wish to characterize contact problems in which the contact surface is approximately planar and the bodies have undergone small deformations in the neighborhood of the contact surface.

Assume the following:

(1) $\underset{\sim}{n} \overset{\text{def}}{=} n_i \underset{\sim}{e}_i \approx \underset{\sim}{e}_3$ on $\mathcal{C}$, where the n_i indicate components with respect to the standard basis $\{\underset{\sim}{e}_i\}_1^3$ for $\mathbb{R}^3$, (see Fig. 5).

(2) $j^\alpha \approx 1$, $\alpha = 1,2$, thus $\underset{\sim}{t}^\alpha \approx \underset{\sim}{T}^\alpha$ on $\mathcal{C}^\alpha$.

Assumptions (1) and (2) together imply that,

$$t_3^\alpha \approx \underset{\sim}{t}^\alpha \cdot \underset{\sim}{n} \approx \underset{\sim}{T}^\alpha \cdot \underset{\sim}{n} \approx T_3^\alpha \quad ,$$

and that,

$$(t_1^\alpha, t_2^\alpha, 0) \approx \underset{\sim}{t}^\alpha - (\underset{\sim}{t}^\alpha \cdot \underset{\sim}{n})\underset{\sim}{n} \approx \underset{\sim}{T}^\alpha - (\underset{\sim}{T}^\alpha \cdot \underset{\sim}{n})\underset{\sim}{n} \approx (T_1^\alpha, T_2^\alpha, 0) .$$

(3) Material points which eventually contact have, to the first order, the same initial coordinates z_1 and z_2. Explicitly we manifest this idea by requiring that the $\underset{\sim}{\chi}^\alpha$'s satisfy

$$\underset{\sim}{\chi}^1(z_1, z_2, X_3^1(z_1, z_2)) = \underset{\sim}{\chi}^2(z_1, z_2, X_3^2(z_1, z_2)) \quad . \tag{39}$$

This is depicted in Fig. 6. Since X_3^α are given functions which define the surfaces $\mathcal{C}^\alpha$, it follows from (39) that,

$$\delta \underset{\sim}{\chi}^1 = \delta \underset{\sim}{\chi}^2 \quad .$$

We term problems for which these assumptions hold Hertzian, since these assumptions are implicit in Hertz' classical theory [2] (see

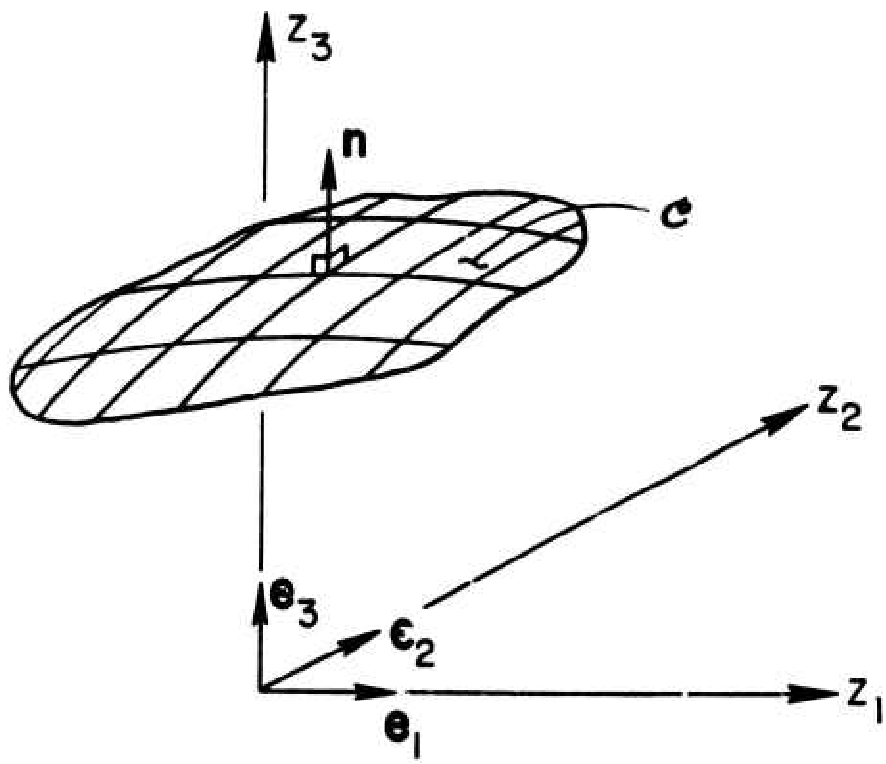


Figure 5

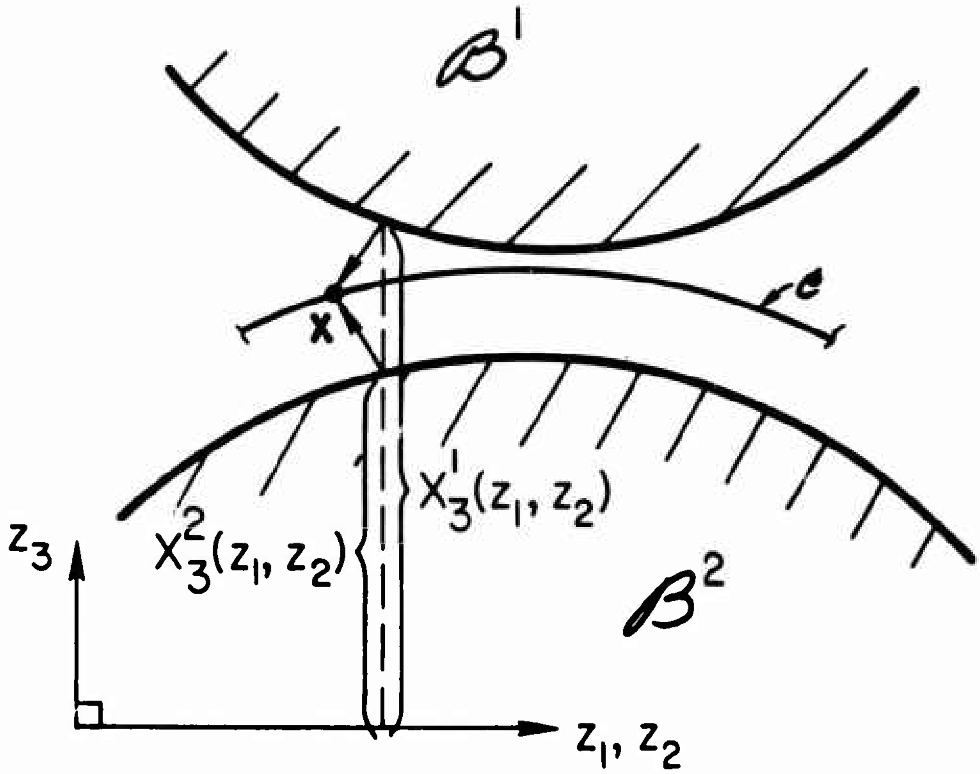


Figure 6

Goldsmith [3] for an excellent exposition of this work and also many applications of Hertz' theory to impact problems). It should be pointed out that the formulation we are about to give is still considerably more general than those to which Hertz' theory applies.

We now show how these assumptions allow us to make simplifications in the preceding theorems.

Theorems I and II:

Due to assumption (3) the term $\mathcal{K}$ can be replaced by an integral over a region in the z_1, z_2 -plane. This region, say c, is the projection of $\mathcal{E}$ onto the z_1, z_2 -plane, and due to assumption (2) it coincides, to the first order, with the projections of the $\mathcal{E}^\alpha$'s. Thus $\mathcal{K}$ can be written

$$\mathcal{K} = - \sum_{\alpha=1}^{2} \int_0^t \int_c \underset{\sim}{\tau}^\alpha \cdot (\underset{\sim}{x}^\alpha - \underset{\sim}{\chi}^\alpha) \, dc \, dt . \tag{40}$$

Since, for Case I, we know that the momentum balance on $\mathcal{E}$ requires that

$$\underset{\sim}{\tau}^1 + \underset{\sim}{\tau}^2 = \underset{\sim}{0} ,$$

we may make use of this relation immediately. Thus define

$$\underset{\sim}{\tau} = \underset{\sim}{\tau}^1 = -\underset{\sim}{\tau}^2 ,$$

and substitute into (40). Employing (39), the integrand simplifies to

$$\underset{\sim}{\tau} \cdot (\underset{\sim}{x}^2 - \underset{\sim}{x}^1) . \tag{41}$$

The analog of (23) becomes

$$0 = \int_0^t \int_C \Big\{ \delta\underset{\sim}{\tau} \cdot (\underset{\sim}{x}^2 - \underset{\sim}{x}^1) \;+$$
$$+ \delta\underset{\sim}{x}^1 \cdot (\underset{\sim}{T}^1 - \underset{\sim}{\tau}) + \delta\underset{\sim}{x}^2 \cdot (\underset{\sim}{T}^2 + \underset{\sim}{\tau}) \Big\}\, dc\, dt$$
$$+ \text{ transversality condition .} \tag{42}$$

Thus the same conclusions of Theorem I can be drawn. However, from a numerical standpoint things are considerably different. First of all, since the $\underset{\sim}{\chi}^{\alpha}$'s are absent in this formulation, we do not get a uniquely defined $\underset{\sim}{t}$; $\underset{\sim}{x}^1$ and $\underset{\sim}{x}^2$ will not in general be the same pointwise. If the graph of $\underset{\sim}{t}$ is important it could be constructed by averaging $\underset{\sim}{x}^1$ and $\underset{\sim}{x}^2$, which, if the solution is any good, should be reasonably close pointwise. On the other hand, the $\underset{\sim}{\chi}^{\alpha}$'s being absent engenders a considerable saving in the number of equations to be solved and in their complexity.

The analogous case for Theorem II is constructed simply by setting

$$\tau_1 = \tau_2 = 0 \quad , \qquad \tau \overset{\text{def.}}{=} \tau_3 \ .$$

Then the integrand of $\mathcal{K}$ becomes

$$\tau\,(x_3^2 - x_3^1) \tag{43}$$

and (42) reduces to

$$0 = \int_0^t \int_C \Big\{ \delta\tau\,(x_3^2 - x_3^1) \;+$$
$$+ \delta x_3^1 (T_3^1 - \tau) + \delta x_3^2 (T_3^2 + \tau)$$
$$+ \delta x_\alpha^1\, T_\alpha^1 \qquad + \delta x_\alpha^2\, T_\alpha^2 \Big\}\, dc\, dt \tag{44}$$
$$+ \text{ transversality condition .}$$

Hence the conclusions of Theorem II hold.

Thus in the case of Hertzian contact we can add the simplifications manifested in (41) and (43) to the conditions of Theorems I and II, respectively, and still garner the same conclusions.

Theorems I' and II':

For these cases $\mathcal{M}$ can be written as an integral over $\tilde{c}$, the projection of $\tilde{\mathcal{E}}$:

$$\mathcal{M} = - \sum_{\alpha=1}^{2} \int_{0}^{t} \int_{\tilde{c}} \underset{\sim}{\sigma}^{\alpha} \cdot n^{\alpha} (\underset{\sim}{x}^{\alpha} - \underset{\sim}{\chi}^{\alpha}) \, d\tilde{c} \, dt \, .$$

Due to the present geometric situation, it is appropriate to take

$$n^{1} = n^{2} \, ,$$

and thus define

$$n = n^{\alpha} \, , \qquad \alpha = 1,2 \, .$$

Analagous to the considerations for Theorems I and II, the momentum balance across $\tilde{\mathcal{E}}$ motivates the simplification

$$\underset{\sim}{\sigma} \overset{\text{def.}}{=} \underset{\sim}{\sigma}^{1} = - \underset{\sim}{\sigma}^{2} \, .$$

With these and (39), the integrand of $\mathcal{M}$ can be written

$$\underset{\sim}{\sigma} \cdot n (\underset{\sim}{x}^{2} - \underset{\sim}{x}^{1}) \, .$$

A further simplification can be made by setting*

$$\sigma_{3} = - n \, .$$

This eliminates one unknown function and, as we shall see, has the effect of satisfying (5) naturally. Thus the integrand of $\mathcal{M}$ becomes

---

* This is a standard ploy of optimization theory, see p. 82, [4].

$$\sigma_\alpha \eta (x_\alpha^2 - x_\alpha^1) - (\eta)^2 (x_3^2 - x_3^1) \ , \tag{45}$$

and the analog of (23) is

$$\begin{aligned} 0 = \int_0^t \int_{\mathcal{C}} \Big\{ & \delta\eta (\sigma_\alpha (x_\alpha^2 - x_\alpha^1) - 2\eta (x_3^2 - x_3^1)) \\ & + \delta\sigma_\alpha (\eta (x_\alpha^2 - x_\alpha^1)) \\ & + \delta x_\alpha^1 (T_\alpha^1 - \eta\sigma_\alpha) + \delta x_\alpha^2 (T_\alpha^2 + \eta\sigma_\alpha) \\ & + \delta x_3^1 (T_3^1 + (\eta)^2) + \delta x_3^2 (T_3^2 - (\eta)^2) \Big\} d\mathcal{C}\, dt . \end{aligned} \tag{46}$$

Summand two tells us that either $\eta = 0$ or $x_\alpha^1 = x_\alpha^2$, on $\mathcal{C}$. Suppose $\eta \neq 0$, then $x_\alpha^1 = x_\alpha^2, \alpha = 1,2$. Summand one then gives us that $x_3^1 = x_3^2$ on $\mathcal{C}$. Thus we have

$$\eta (\underset{\sim}{x}^2 - \underset{\sim}{x}^1) = \underset{\sim}{0} \ , \qquad \text{on} \quad \mathcal{C} \ ,$$

as required, and $\mathcal{e}$ is defined as the subset of $\mathcal{C}$ where $\underset{\sim}{x}^1 = \underset{\sim}{x}^2$.

The last four summands give the momentum balance conditions, as usual, and, in addition, the last two summands imply that the normal tractions are compressive (since $(\eta)^2 \geq 0$). Thus we have the conclusions of Theorem I' and condition (5).

The analogous set up for Theorem II' is accomplished by setting $\sigma_\alpha = 0$ in (45) yielding

$$-(\eta)^2 (x_3^2 - x_3^1) \tag{47}$$

for the integrand of $\mathcal{M}$. With this Eq. (46) becomes

$$0 = \int_0^t \int_{C} \Big\{ -2\,\delta\eta\big(\eta\,(x_s^2 - x_s^1)\big) + \delta x_\alpha^1 T_\alpha^1 + \delta x_\alpha^2 T_\alpha^2 + \delta x_s^1\big(T_s^1 + (\eta)^2\big) + \delta x_s^2\big(T_s^2 - (\eta)^2\big)\Big\}\, d\tau\, dt$$

In this case we achieve the conclusion of Theorem II' and condition (5).

Thus to Theorems I' and II' we can delete condition (5), add the simplifications manifested in (45) and (47), and achieve the conclusions of Theorems I' and II', respectively, plus condition (5).

6. Contact Problems for One, Two and Three-dimensional Bodies

The previous work needs only trivial modification to be made applicable to contact problems involving bodies of different dimensions. There are many cases of considerable interest which fall into this category. For example, models consisting of a shell and a plate, or a solid and a plate, are useful for the study of head impact. The modifications necessary are essentially interpretative. An example illustrates this assertion.

Consider the frictionless Hertzian contact of a three-dimensional solid and a two-dimensional plate. Let $\mathcal{B}^1$ represent the solid and $\mathcal{B}^2$ the plate. In evaluating Π, the $\mathcal{B}^1$ part is as before while the $\mathcal{B}^2$ part would manifest the particular plate theory used. The contact term $\mathcal{K}$ (or $\mathcal{M}$) would be exactly as before. However note that c (or $\bar{c}$) is, in this case, also identifiable with part of the two-dimensional "volume" of the plate, rather than its boundary. Taking variations, everything is as before except that the term $\tau \delta x_3^2$ (or $-(\eta)^2 \delta x_3^2$) contributes to the transverse momentum equation of the plate, rather than to its boundary conditions. The interpretation of τ (or $-(\eta)^2$) is thus two-fold, i.e., it is the normal component of the traction vector with respect to $\mathcal{B}^1$, as before, and it is also the equivalent normal "body force" with respect to $\mathcal{B}^2$, manifested by the interaction with $\mathcal{B}^1$ (Fig. 7).

This interpretation is general, namely, for one and two-dimensional bodies the contact force is an equivalent "body force" which contributes to the momentum equations, rather than the boundary conditions. With this interpretation in mind, the construction of variational theorems, analogous to the ones constructed in Sections 2, 4 and 5, for the class of one, two and three-dimensional contact problems, is just a formal deductive exercise involving only appropriate definitions for Π.

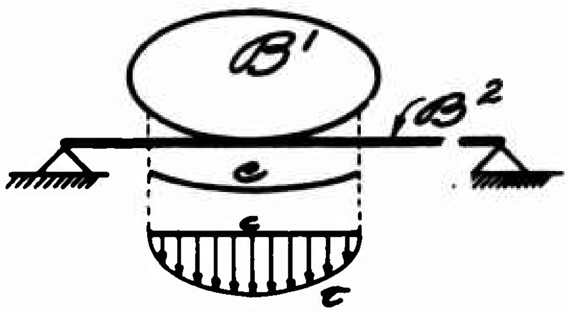


Figure 7

7. Impact

The previous sections deal with spatial aspects of contact problems. In this section we investigate temporal considerations, i.e., those phenomena which are unique to dynamic contact or impact. To manifest the problem encountered in such situations consider the following hypothetical situation. Assume that we are in the process of numerically solving some impact problem and suppose that it is discovered as we monitor the motion of the bodies that they impact somewhere in the time interval (t_1,t_2). At time t_1 we know the states of both bodies and we know that somewhere between t_1 and t_2 they have coalesced over a portion of their boundaries. Assume for the moment we know the geometry of the contact surface $\mathfrak{C}$. The question which arises then is what is the state of $\mathfrak{C}$ at time t_2, i.e., what are the velocity and traction vectors on $\mathfrak{C}$? It is necessary to know this information to carry forth the step forward time integration. The question though seems improperly posed without specifying considerable data about the nature of the impact. To get a handle on things, we will initially formulate a simple one-dimensional problem involving the impact of two elastic rods. Although this problem is trivial, it provides considerable insight into the general nature of impact of continuum bodies. Since we are interested in the state of $\mathfrak{C}$ (in this case a point) immediately after impact, whether the rods are finite or semi-infinite is immaterial.

Assume that the pre-impact states of the two bodies are given by the following data:

$$\mathcal{B}^\alpha \quad v_-^\alpha , \quad (\partial x/\partial X)_-^\alpha , \quad P_-^\alpha \quad ; \alpha = 1,2 . \tag{48}$$

At impact the rods coalesce at $\mathfrak{e}$, and for some finite time interval thereafter (at least) $x \in \mathfrak{e}$ is persistent. At the moment of impact shock waves begin to propagate in each body. The space-time picture is depicted in Fig. 8. As discussed in section 1, since $\mathfrak{e}$ is material and x is persistent, we have

$$v \overset{def}{=} v^1_+ = v^2_+ \quad , \qquad P \overset{def}{=} T^1_+ = -T^2_+ \quad , \tag{49*}$$

for the post-impact state (t_+). In addition to (49), the well known shock conditions must hold across the wave fronts:

$$\begin{aligned} &[v^\alpha] + U^\alpha[(\partial x/\partial X)^\alpha] = 0 \; , \\ &\rho_0^\alpha U^\alpha[v^\alpha] = [P^\alpha] \; , \end{aligned} \tag{50}$$

where U^α is the material velocity of the shock in $\mathcal{B}^\alpha$, and [ ] is the wave-front jump operator which assigns to a function the difference in its values behind and in front of the wave, i.e., $[f(X,t)] = f(X^-,t) - f(X^+,t)$ where X is a material point denoting the location of the wave-front. As can be deduced from Fig. 8, the states into which the shocks initially propagate are the pre-impact states given by (48), and the state at $\mathfrak{e}$, immediately after the shocks pass, is given by the post-impact state (49). These observations in conjunction with (50) yield,

$$\begin{aligned} &v^\alpha_- - v + U^\alpha\{(\partial x/\partial X)^\alpha_- - (\partial x/\partial X)^\alpha_+\} = 0 \; , \\ &\rho_0^\alpha U^\alpha(v^\alpha_- - v) + P^\alpha_- - P = 0 \; . \end{aligned} \tag{51**}$$

*For convenience we choose the initial state to be the pre-impact state, thus we need not distinguish between Cauchy and Piola tractions.

**A consistency condition for these equations is that $v^1_- - v^2_- > 0$. Otherwise the impact would not occur.

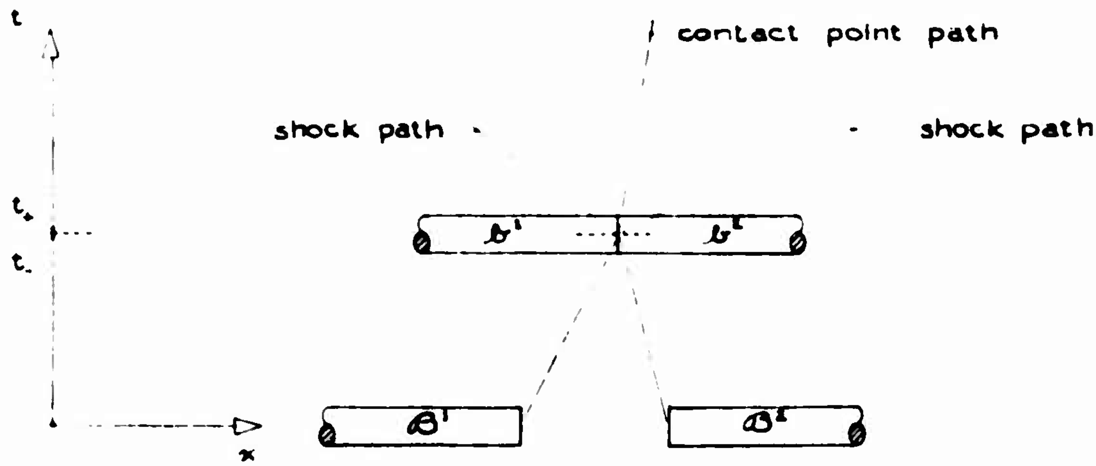


Figure 8

The four Eqs. (51) and constitutive equations relating P^{α} to $(\partial x/\partial X)^{\alpha}$ yield a formally deterministic system of six equations in the six unknowns $v, P, U^{\alpha}, (\partial x/\partial X)^{\alpha}$. Thus we see that the desired quantities v and P depend on the pre-impact states and material properties of both $\mathcal{B}^{\alpha}$. The precise form of this relationship depends upon the constitutive equations of the bodies. As a simple example, assume we have linear constitutive equations $P^{\alpha} = E^{\alpha}\{(\partial x/\partial X)^{\alpha} - 1\}$, E^{α} constant, and let the pre-impact state be given by

$$
\begin{aligned}
v_-^{\alpha} &= V^{\alpha} , \\
(\partial x/\partial X)_-^{\alpha} &= 1 , \\
P_-^{\alpha} &= 0 .
\end{aligned}
\tag{52}
$$

These conditions, when inserted in Eqs. (51), lead to:

$$
\begin{aligned}
v &= \frac{\rho_o^2 U^2 V^2 - \rho_o^1 U^1 V^1}{\rho_o^2 U^2 - \rho_o^1 U^1} , \\
P &= \frac{V^2 - V^1}{\left(\frac{U^2}{E^2} - \frac{U^1}{E^1}\right)} , \\
(U^{\alpha})^2 &= E^{\alpha}/\rho_o^{\alpha} .
\end{aligned}
\tag{53}
$$

Note that the denominators in Eqs. $(53)_{1,2}$ present no problems since $\rho_o^{\alpha} > 0$, $E^{\alpha} > 0$ and $+1 = \mathrm{sgn}\, U^2 = -\,\mathrm{sgn}\, U^1$.

This result is also appropriate whenever the intensity of the impact is small enough such that the non-linear constitutive equation can be replaced by its linear approximation about the pre-impact state. In this case E^{α} is a tangent modulus evaluated at the pre-impact strain $\{(\partial x/\partial X)_-^{\alpha} - 1\} = 0$. To further simplify, consider the case when both rods have identical properties (i.e., $\rho_o = \rho_o^{\alpha}$, $E = E^{\alpha}$, $\alpha = 1, 2$). Then

$$v = \frac{V^1 + V^2}{2} ,$$

$$P = \rho_0 U (V^2 - V^1)/2 , \qquad (54)$$

$$(U)^2 = E/\rho_0 .$$

In Eqs. (54) U is positive, and since consistency requires $V^1 - V^2 > 0$, P is compressive.

Thus for the one-dimensional case at least the problem of computing the post-impact state is easily achieved. The solution of (51) for the fully non-linear case can be automated as part of a numerical algorithm. Although this problem is trivial, it serves to indicate that the post-impact problem, the solution of which is essential in a numerical algorithm, is one of wave propagation.

In the analysis of higher dimensional bodies the solution of the post-impact problem becomes greatly complicated due to the geometric variety of impact conditions. However, considerable simplifications can be taken advantage of if one keeps in mind the nature of the discrete problem. For instance, if a certain portion of the boundaries of two bodies have coalesced in $\mathfrak{c}$, each interior point of $\mathfrak{c}$, at which the tangent plane is well defined, may be treated, to the first order, as a point on the mating surface of two impacting half-spaces. As long as time steps are kept small enough, the local behavior is well represented. The post-impact problem for the general case, analogous to (51), can be automated as part of the numerical algorithm, and for many simple cases can be solved explicitly.

With these notions in mind, let us return to the case of main interest in this report, namely three-dimensional continuum bodies. We shall consider only the case of a frictionless contact surface (Case II), and leave the solution of the post-impact problem for the no-slip case (Case I), which is more difficult, for future work. With the proper interpretations, the one-dimensional rod formulation (Eqs. (48-54)) suffices to completely characterize this case. This is so because no tangential motions or stresses may be communicated across a frictionless surface, and thus we need only consider the configuration of normal incidence. In this case the requisite constitutive functionals in (51) would be those relating P^{α}, the normal Piola stress, to the normal component of strain, holding all other components of strain fixed at the pre-impact values. For example, in the linear isotropic case, E^{α} (Young's modulus) in Eqs. (53,54) would be replaced by $\lambda^{\alpha}+2\mu^{\alpha}$ ($\lambda^{\alpha},\mu^{\alpha}$ are the Lamé and shear moduli, respectively) and the propagation velocity would be that of dilatational waves.

PART II

A NUMERICAL SCHEME FOR ANALYSIS OF CONTACT-IMPACT PROBLEMS

8. Numerical Solution of Contact-Impact Problems

In performing numerical computations based on the above described variational formulation for contact-impact problems we have employed three distinct levels of approximation: (1) a spatial discretization of the bodies and contact surfaces, (2) a temporal discretization to determine the response of the discretized bodies, and (3) a numerical solution for the resulting system of nonlinear algebraic equations.

In the following sections we shall restrict our attention to the Hertzian contact problem described in Section 5. Significant numerical difficulties are encountered in the solution of impact problems; to complicate the problem further by introducing the additional steps necessary to determine the contact surface maps for the full kinematically nonlinear case is left for a future study. While this is a simple impact problem in terms of determining the contact surface and the full power of the preceding theory is neither necessary nor exploited in its solution, many of the features of the general problem are employed here.

9. Spatial Discretization of the Bodies and Contact Surface

The bodies $\mathcal{B}^1$ and $\mathcal{B}^2$ are discretized using standard finite element methods, (e.g., see [5]). In order to facilitate the computation of a discrete Hertzian contact surface the nodes of $\mathcal{B}^1$ are arranged so that they align with the nodes of $\mathcal{B}^2$. This is consistent with the notions of condition 3 of Section 5 and ensures that during determination of the approximation to the contact surface contiguous nodes of the two bodies will meet. Thus, the simulation of the contact surface is trivial. The development of a numerical model for Hertzian contact problems is based upon the form of Theorem II' which uses (47) for the integrand of $\mathcal{M}$. For numerical computations we introduce the displacement vector $\underset{\sim}{u}$ such that

$$\underset{\sim}{x} = \underset{\sim}{X} + \underset{\sim}{u} \quad . \tag{55}$$

For a compatible finite element displacement field the integrand of $\mathcal{M}$ can be approximated by taking $\eta^2(\underset{\sim}{z},t)$ as the product of $\mathcal{E}^2(t)$ and $\delta(\underset{\sim}{z}-\underset{\sim}{z}_i)$ (i.e., Dirac delta functions in space). This corresponds to taking η^2 as "concentrated nodal loads" which are the generalized forces of the contact pressure. With this discretization we can describe pseudo contact elements between each pair of candidate contact nodes. Let these nodes be denoted as $(\ )_i^\alpha$ and the generalized force as $(\mathcal{E}_i)^2$; then

$$\mathcal{M} = \int_0^t \sum_i (\mathcal{E}_i(t))^2 (u^2_{3i}(t) - u^1_{3i}(t) + X^2_{3i} - X^1_{3i})dt \tag{56}$$

where $\{i\}$ are the set of candidate contact nodes which span $\tilde{c}$; u^α_{3i} are the nodal displacements in the z_3 direction and X^α_{3i} are the nodal coordinates of the candidate contact nodes.*

*We assume here that 3 is the direction nominally normal to the contact surfaces, e.g., see Fig. 5.

Use of the finite element method in Theorem II' with $\mathcal{m}$ given by (56) produces a set of nonlinear second order ordinary differential equations which together with the impenetrability conditions define the discretized contact impact problem. These equations take the form:

$$\underset{\sim}{M}\,\ddot{\underset{\sim}{u}} + \underset{\sim}{K}(\underset{\sim}{u}) = \underset{\sim}{R} \quad , \tag{57}$$

where $\underset{\sim}{M}$ is the usual finite element mass matrix, $\underset{\sim}{K}$ represents the elastic stiffness forces together with the contact terms, $\underset{\sim}{R}$ is the set of generalized forces resulting from boundary tractions and $\underset{\sim}{u}$ is the set of time dependent nodal displacements (which also include the $(\varepsilon_i)^2$). For inelastic materials Theorem II' can be extended by treating the first variation as a Galerkin method (principle of virtual work) and replacing the elastic constitutive model by more general theories, e.g., viscoelastic, elasto-plastic, viscoplastic, etc. In this case

$$\underset{\sim}{K}(\underset{\sim}{u}) \longrightarrow \underset{\sim}{K}(\underset{\sim}{u},\dot{\underset{\sim}{u}}) \tag{58}$$

in (57).

10. Temporal Discretization

A temporal discretization of the second order ordinary differential equations which result from a finite element spatial discretization of the contact-impact problem is accomplished herein by using the Newmark family of methods [6]. The Newmark family of methods is a one-step integration method with two free parameters which can be used to control stability and numerical damping. The method is essentially a difference method in time. The behavior of the method for linear elasto-dynamics problems is discussed in [6,7]. The algorithm is given by

$$\underset{\sim}{u}_{n+1} = \underset{\sim}{u}_n + \Delta t \, \dot{\underset{\sim}{u}}_n + (\tfrac{1}{2} - \beta)\Delta t^2 \ddot{\underset{\sim}{u}}_n + \beta \Delta t^2 \ddot{\underset{\sim}{u}}_{n+1} \, , \tag{59}$$

and
$$\dot{\underset{\sim}{u}}_{n+1} = \dot{\underset{\sim}{u}}_n + (1-\gamma)\Delta t \, \ddot{\underset{\sim}{u}}_n + \gamma \Delta t \, \ddot{\underset{\sim}{u}}_{n+1} \, ,$$

where $\underset{\sim}{u}_n = \underset{\sim}{u}(t_n)$, $\Delta t = t_{n+1} - t_n$, and β, γ are the two parameters. For linear problems $\gamma = .5 + \delta = .5$ produces no artificial viscosity and $\beta \geq \frac{1}{4}(1+\delta)^2$ produces unconditional stability (i.e., the method is stiffly stable). Such generalization is not possible for nonlinear problems and during solution it may be necessary to monitor the solution for any signs of instability. In (59) $\beta = 0$ produces an explicit method for $\underset{\sim}{u}_{n+1}$ and if $\underset{\sim}{M}$ is diagonal (lumped mass) with $\underset{\sim}{K}$ and $\underset{\sim}{R}$ independent of $\dot{\underset{\sim}{u}}$ the solution can be advanced without solving a large set of simultaneous equations; for all other cases the method is implicit and equations must be solved. Solution of $(59)_1$ for $\ddot{\underset{\sim}{u}}_{n+1}$ in terms of the solution at t_n and $\underset{\sim}{u}_{n+1}$ gives

$$\ddot{\underset{\sim}{u}}_{n+1} = \frac{1}{\beta \Delta t^2}(\underset{\sim}{u}_{n+1} - \underset{\sim}{u}_n) - \frac{1}{\beta \Delta t}\dot{\underset{\sim}{u}}_n - \left(\frac{1-2\beta}{2\beta}\right)\ddot{\underset{\sim}{u}}_n \tag{60}$$

which can also be used in $(59)_2$ to express the velocity in terms of the solution at t_n and $\underset{\sim}{u}_{n+1}$. Since in this process we divide by β and Δt it is no longer possible to consider zero β or zero time steps.

11. Solution of the Nonlinear Algebraic Problem

Use of the Newmark method in (57) (including (58)) yields the set of nonlinear algebraic equations:

$$\frac{1}{\beta\Delta t^2}\underset{\sim}{M}\,\underset{\sim}{u}_{n+1} + \underset{\sim}{K}(\underset{\sim}{u}_{n+1}, \underset{\sim}{u}_n, \dot{\underset{\sim}{u}}_n, \ddot{\underset{\sim}{u}}_n) = \underset{\sim}{R}_{n+1} + \underset{\sim}{M}\,\underset{\sim}{A}_n \quad , \tag{61}$$

where

$$\underset{\sim}{A}_n = \frac{1}{\beta\Delta t^2}\,\underset{\sim}{u}_n + \frac{1}{\beta\Delta t}\,\dot{\underset{\sim}{u}}_n + \left(\frac{1-2\beta}{2\beta}\right)\ddot{\underset{\sim}{u}}_n \quad .$$

A Newton-Raphson iterative solution to this set of equations can formally be constructed, giving:

$$\left(\frac{1}{\beta\Delta t^2}\underset{\sim}{M} + \partial_u\underset{\sim}{K} - \partial_u\underset{\sim}{R}\right)\Delta\underset{\sim}{u}^{(i)} = \underset{\sim}{R} - \underset{\sim}{K}(\underset{\sim}{u}^{(i)}_{n+1}, \underset{\sim}{u}_n, \dot{\underset{\sim}{u}}_n, \ddot{\underset{\sim}{u}}_n) - \underset{\sim}{M}\,\ddot{\underset{\sim}{u}}^{(i)}_{n+1} \quad , \tag{62}$$

where $\partial_u \underset{\sim}{R}$ is the effect of loads varying with the deformation and

$$(\partial_u\underset{\sim}{K})_{ij} = \partial K_i / \partial u_j \quad , \tag{63}$$

is the tangent stiffness matrix. The coefficient to $\Delta\underset{\sim}{u}^{(i)}$ is generally called the Jacobian matrix of the Newton-Raphson iteration. The solution is advanced by taking

$$\underset{\sim}{u}^{(i+1)}_{n+1} = \underset{\sim}{u}^{(i)}_{n+1} + \Delta\underset{\sim}{u}^{(i)} \quad , \tag{64}$$

and iterating until a norm of the solution satisfies

$$\|\Delta\underset{\sim}{u}^{(i)}\| \leq \epsilon\,\|\underset{\sim}{u}^{(i)}_{n+1}\| \quad , \tag{65}$$

where ϵ is some small positive error tolerance. In the work reported here the norm $\| \ \|$ is taken as the Euclidian norm

$$\| \underset{\sim}{x} \| = \left(\sum_{i} x_i^2 \right)^{1/2} , \tag{66}$$

and the load vector $\underset{\sim}{R}$ is assumed to be independent of $\underset{\sim}{u}$. For stable elastic materials the resulting tangent stiffness is then symmetric and positive definite, consequently, standard direct solution methods normally employed in the solution of linear finite element problems can be used. For inelastic materials or deformation dependent loads the tangent stiffness may be asymmetric. In these cases some special methods may be necessary to effect a solution.

12. Discretized Impact Conditions

In the previous numerical development $\tilde{c}$ has been defined by discrete points which correspond to nodes along the boundaries of $\mathcal{B}^1$ and $\mathcal{B}^2$. When, during the course of advancing the solution in time, any one of these points violates the impenetrability condition a re-solution must be obtained in which the $(\varepsilon_i)^2$ are now non-zero and the u_3^* satisfy the impenetrability condition. Some control and monitoring are required to effect this in a computer program. In addition to satisfying these conditions, the impact relations denoted in Section 7 must be invoked. In the present study these conditions are applied to the solution at the end of a time step in which points first go into contact. Accordingly we compute from (50)*

$$\dot{u}_+ = \frac{\rho_0^2 U^2 \dot{u}_-^2 - \rho_0^1 U^1 \dot{u}_-^1}{\rho_0^2 U^2 - \rho_0^1 U^1} , \qquad (67)$$

and assign this value to the appropriate node of $\mathcal{B}^1$ and $\mathcal{B}^2$.

To determine the solution vector $\underset{\sim}{u}$ at t_{n+1} we have solved the set of equations (61). As described above the shock conditions are then used to determine the value of the velocity at time t_{n+1} for all points which have come into contact during the time interval. In order to get a consistent solution at these points we must modify the accelerations and contact force to reflect the shock conditions. This is accomplished by re-solving the equilibrium conditions of $\mathcal{B}^1$ and $\mathcal{B}^2$ at point i. The expanded forms of the appropriate equations are:

---

*The $(\ )_-$ denotes a value which is computed before impact, whereas $(\ )_+$ denotes the value after impact.

$$M^1\ddot{u}^1_- + K^1(\underset{\sim}{u}) + (\varepsilon_i)^2_- = R^1 , \tag{68}$$

and

$$M^2\ddot{u}^2_- + K^2(\underset{\sim}{u}) + (\varepsilon_i)^2_- = R^2 .$$

For nodes which have come into contact we must enforce the condition on acceleration

$$\ddot{u}^1_+ = \ddot{u}^2_+ = \ddot{u}_+ , \tag{69}$$

and compute the contact force $(\varepsilon_i)^2{}_+$. The solution for these is obtained from

$$M^1\ddot{u}_+ + K^1(\underset{\sim}{u}) + (\varepsilon_i)^2_+ = R^1 ,$$

and

$$M^2\ddot{u}_+ + K^2(\underset{\sim}{u}) + (\varepsilon_i)^2_+ = R^2 .$$

These are two equations in two unknowns which can be solved for the $\ddot{u}_+$ and $(\varepsilon_i)^2_+$. If $K^\alpha(\underset{\sim}{u})$ is independent of velocity the stiffness forces and R^α will remain unchanged during the impact, hence we can solve the simpler problem

$$M^1\ddot{u}_+ + (\varepsilon_i)^2_+ = M^1\ddot{u}^1_- + (\varepsilon_i)^2_-$$

$$M^2\ddot{u}_+ - (\varepsilon_i)^2_- = M^2\ddot{u}^2_- - (\varepsilon_i)^2_-$$

whose solution is

$$\ddot{u}_+ = \frac{M^1\ddot{u}^1_- + M^2\ddot{u}^2_-}{M^1 + M^2} ,$$

and

$$2(\varepsilon_i)^2_+ = 2(\varepsilon_i)^2_- + M^1(\ddot{u}^1_- - \ddot{u}_+)$$
$$- M^2(\ddot{u}^2_- - \ddot{u}_+) .$$

This completes the numerical specification of the solution at t_{n+1}; this solution process is now repeated for each of the succeeding time steps.

At this point it is important to compare the solution procedure for impact of a continuum discretized by a finite element method with the solution procedure for a physically discrete body, i.e., a body composed of mass points joined by massless elastic springs. Both problems may be described by algebraic equations of the form of (57). The impenetrability condition is also identical. The impact conditions, however, are different. For the discretized continuum the procedure is described above. The study of the impact of mass points is considered in elementary mechanics books, e.g. [8]. The impact of two mass points is described by impulsive motion such that at t_- the velocities of the two mass points are V_-^1 and V_-^2; after impact at time t_+, the two points have velocities V_+^1 and V_+^2. The two points will not in general stay in contact (i.e., $V_+^1 \neq V_+^2$) but will rebound. The conditions used to compute the V_+^1 and V_+^2 are:

Balance of Momentum*

$$M^1\{V^1\} + M^2\{V^2\} = 0 , \tag{71}$$

and use of an equation involving the "coefficient of restitution", e :

$$\frac{V_+^2 - V_+^1}{V_-^1 - V_-^2} = e . \tag{72}$$

For $e=1$ energy is conserved whereas for $e=0$ the points "stick" and energy is dissipated. We must comment in passing that (72) is the energy

* $\{f(t)\} = f(t_+) - f(t_-)$.

equation in disguise. To see this we can write the jump conditions for energy as

$$\frac{1}{2} M^1 \{(V^1)^2\} + \frac{1}{2} M^2 \{(V^2)^2\} = \{\nu\} \quad . \tag{73}$$

The term $\{\nu\}$ can exist only if other energies are dissipated during the jump. We rewrite (73) by using

$$\frac{1}{2} \{(V^1)^2\} = [V^1]\langle V^1 \rangle \quad ,$$

where

$$\langle V^1 \rangle = \frac{1}{2}(V_+^1 + V_-^1) \quad . \tag{74}$$

Use of the momentum equation (71) then gives, after dividing by $M^1\{V^1\}$

$$\langle V^1 \rangle - \langle V^2 \rangle = \frac{\{\nu\}}{M^1\{V^1\}} \quad ,$$

or after recollecting terms and dividing by $(V_-^1 - V_-^2)$ we obtain:

$$\frac{V_+^2 - V_+^1}{V_-^1 - V_-^2} = 1 - \frac{\{\nu\}}{M^1\{V^1\}(V_-^1 - V_-^2)} \quad . \tag{75}$$

The significance of the coefficient of restitution then is associated with the right hand side of (75).

It is clear from the above developments that the numerical simulation of the discretized continuum and the physically discrete system involve two distinct methods for treating the impact conditions. It is imperative then to associate the correct method for the problem at hand. In the

present study we are interested in the impact of continua, and in this case we shall employ the discrete shock condition to effect the solution. This a priori assumes that the response we are computing involves a time scale associated with wave propagation problems. Consequently, we cannot expect the computation procedure for advancing the solution in time to be accurate if we take time steps greatly in excess of transit times through each body. In this context it may be important to consider an "explicit" time integration procedure in future work. The stability restrictions may be too severe to make this feasible.

PART III

FEAP 74 - A COMPUTER PROGRAM FOR SOLUTION OF CONTACT-IMPACT PROBLEMS

13. Development of a Contact-Impact Model for FEAP

In order to incorporate an ability to compute solutions to contact-impact problems using a finite element method as described above it is necessary to have available a computer program which can solve the nonlinear equations of motion given by (61). The computer program FEAP is a general program to solve finite element problems. The program has a capability of solving both quasistatic and dynamic problems and can incorporate several types of elements simultaneously. The nonlinear capabilities required for the solution of contact-impact problems have been incorporated into FEAP and currently includes the user options (see Input Instructions):

(1) Selection of quasistatic or dynamic option: The dynamic option will integrate the equations of motion using the one-step Newmark method to advance the solution in time. Quasistatic analysis is accommodated by any one-step algorithm. The algorithm employed is incorporated into each element routine and thus id defined by the developer of each element. Impact problems require description of the contact surface and wave speeds.

(2) Selection of the nonlinear method to advance the solution: Options include:

  (a) No iterations in each time step. Unbalanced forces at each time are added to the next time step.

  (b) Iterations in each time step to achieve a balance of force within each time step. In this option the user can select to reform

the Jacobian matrix for each iteration or only at the first iteration in each time step.

In the impact problems solved to date it has been necessary to use the general form of the Newton-Raphson algorithm. This includes a complete formating and factoring of the Jacobian matrix for each iteration of each time step in the analysis. If the method described herein is to become computationally effective improvements in the computer program are paramount. Undoubtedly the most important aspect in reducing computer times is to introduce a substructuring system so that the highly nonlinear equations in the vicinity of the contact surface can be isolated from the remainder of the bodies. This will normally involve only a small number of equations in the total system of (62). The solution of a large finite element problem will generally concentrate the computer solution time in the forming and factoring of the tangent stiffness matrix. The fewer times that it is necessary to perform this costly step the more efficient tne solution algorithm. Substructuring can be used then to restrict the part of the equations which must be formed and factored often, and thus greatly reduce the computer costs in analyzing impact problems.

The version of FEAP which can currently be used to analyze contact-impact problems includes, in addition to the nonlinear Newton-Raphson iterative algorithm, a new special contact-impact element and a new subroutine to describe impact surfaces and the discrete shock conditions described in Section 12. These are described in the following sections.

14. Contact Element for Hertzian Contact

The contact-impact element which has been developed is called ELMT05 and can be used along any coordinate direction. As developed it cannot be used along normals which are in non-coordinate directions. The development of the contact element assumes that within the framework of linear elasticity theory a node on ω^1 will impact on a node of ω^2. In using this contact element we shall assume that the contact surface on ω^2 is located at larger coordinate values than the contact surface of ω^1. The contact element is described by three nodes. Node 1 is associated with ω^1, Node 3 is associated with ω^2, and Node 2 is used as storage for the contact force $(\mathcal{E})^2$. The user can select the direction of contact motion by specifying the degree of freedom of the nodal unknowns to which the contact is to be measured; this must agree with the physical direction of the element (see Fig. 9). The degree of freedom for the contact element is specified during the MATERIAL data input and consists of a single card in I5 format. The element nodes are described along with all other elements according to Section 4 of the Input Instructions. The Node order as shown in Fig. 9 must be observed.

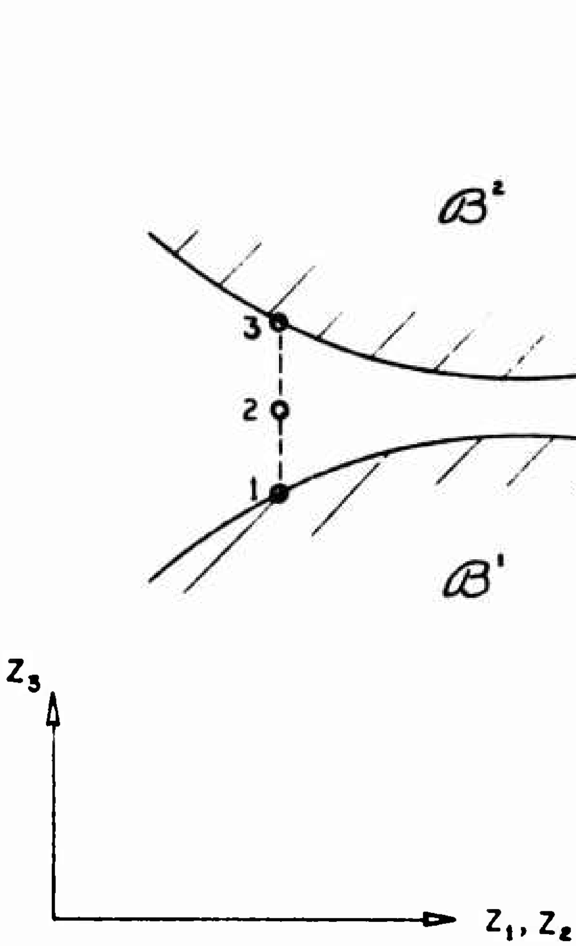


Figure 9

15. Impact Surface Description

The definition of the impact surface includes a list of all elements on the contact surface together with the degree of freedom describing the direction of contact motion (as described above). In addition, the product of mass density and wave velocity (always a positive number) for each body is input. This assumes, currently, that (1) each contact surface belongs to a linear material, and (2) the same material exists along all of the contact surface. This data need be prescribed only for impact problems, quasistatic contact problems do not require this data since no velocity or acceleration computations are performed in this class of problems. Data to be input for the impact surface is given in Table I.

Table I - Impact Surface Data

CARD 1) (6X,A6)

COL. 7 to 12		Must contain CONTAC

CARD 2) (2F10.0)

COL. 1 to 10	ρU	of body 1
COL. 11 to 20	ρU	of body 2

CARD 3) (I5)

COL. 1 to 5		NLIST, number of elements on contact surface

CARD 4) (2I5)

Repeat NLIST times

COL. 1 to 5		Contact element number
COL. 6 to 10		Degree of freedom of this contact element

16. Example Problems

Two example problems are included to illustrate the characteristics of the methodology and the associated computer program described above for Hertzian contact problems. The first problem is a quasistatic contact problem which is used to demonstrate the ability of the computer program to compute an evolving contact surface. The second problem will demonstrate the ability of the program to properly model the temporal response of an impact problem.

To model a problem in which a contact surface will change under different load levels we consider two beams with an initial parabolic curvature. A symmetric configuration is analyzed and the resulting finite element model is shown in Fig. 10. Each element is nominally one unit by one unit. The gap at the load end is initially 0.5 units. The material properties used are $E = 500$ and $\nu = 0$. The load P is applied as shown and allowed to increase linearly in time. The problem then is to determine the contact surface at various load levels. In order to eliminate a singularity in the system of equations it was necessary to permanently attach the two nodes at the symmetry axis of the contact surface. All other nodes along the boundaries between the two bodies are assumed to be possible cont-ct points and contact elements are assigned between vertical nodal pairs. The load was varied from 0.2 to 0.8 in increments of 0.1 and the computed contact surface and forces were computed. These are given in Table II. The deformed shape at a load of 0.4 is also shown in Fig. 10 as a dotted form. The attached node at the center has influenced the solution at loads above 0.3 since the contact pressure there is tensile (negative). The force is small and should not greatly affect the actual contact region computed. As the load increases the

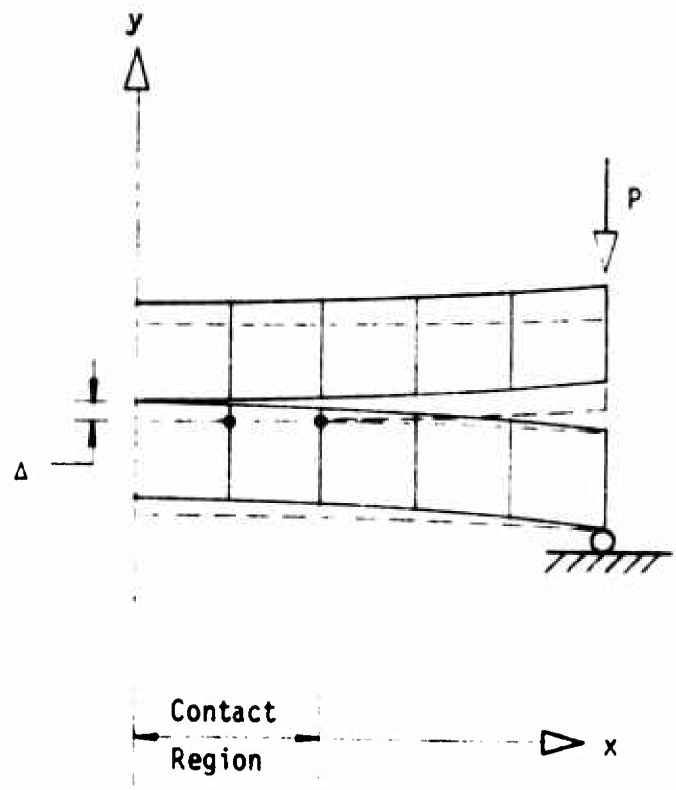


Figure 10

contact surface moves toward the load. This is conceptually correct since if the beams were modeled according to Euler-Bernoulli theory the contact force would be a point load which gradually moves from the center to the outer edge according to the relationship (using the above values for sizes and material properties)

$$X = 5\left(1 - \frac{1}{6P}\right) \quad .$$

This relation predicts that the contact point will be non-zero only after P exceeds 1/6. The finite element model is in qualitative agreement with this beam theory, but since shear deformations are included the finite element solution gives a distributed load on the contact surface. It is interesting to also note that the contact force over the center of the beams is zero, just as in the beam theory.

Table II - Contact Forces

	X-COORDINATE						BEAM
LOAD	0	1	2	3	4	5	THEORY-X
0.2	0.2	-	-	-	-	-	0.83
0.3	.07	.23	-	-	-	-	2.22
0.4	-.01	.07	.34	-	-	-	2.92
0.5	-0.00	.02	.23	.25	-	-	3.33
0.6	-.01	-	.09	.51	.01	-	3.61
0.7	-.01	-	.07	.45	.19	-	3.81
0.8	-.01	-	.05	.38	.38	-	3.96

This problem demonstrates that the computer program can model the evolution of a contact surface. Of particular importance is to note that as the load increases the program can both attach and detach a contact point. This is an essential requirement for the analysis of

impact problems as is shown in the next problem.

As a simple example we consider the impact against a rigid wall of a finite, linear elastic rod traveling at constant velocity. The rod has a modulus of elasticity E of 100, and a mass density ρ of 0.1. The arrival velocity is taken to be 0.1 (units may be assigned in any convenient system). The rod is taken to be 10 units long and is divided into 10 elements plus one contact element as shown in Fig. 11. At time zero the rod is just arriving at the wall. The exact solution predicts a contact duration of 0.2 time units. This corresponds to the time required for a wave to travel from the contact point to the left end and back to the contact point at which time the rod will part from the wall. The problem was analyzed using FEAP with time steps of 0.01 unit (transit time across an element) and the rod remains in contact until time 0.20 units and has rebounded at time 0.21. Thus the program can predict accurately the contact duration of the rod. The finite element solution obtained is compared with the exact solution in Fig. 12. The agreement of stresses and contact force is good. The largest discrepancy exists in defining the shock front, which is "smeared" by the finite element method and ordinary differential equation solution method used here. This is the same type of solutions which are commonly obtained with numerical solutions of this type even without impact. Solutions such as the impact shocks generated are probably one of the most difficult responses to accurately calculate by a finite element method.

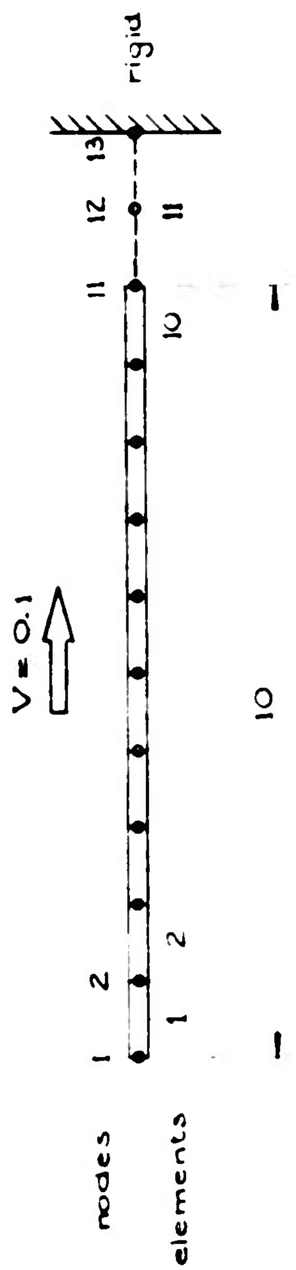


Figure 11

Comparison of Numerical Data with Exact Solution for Bar Impacting Rigid Wall

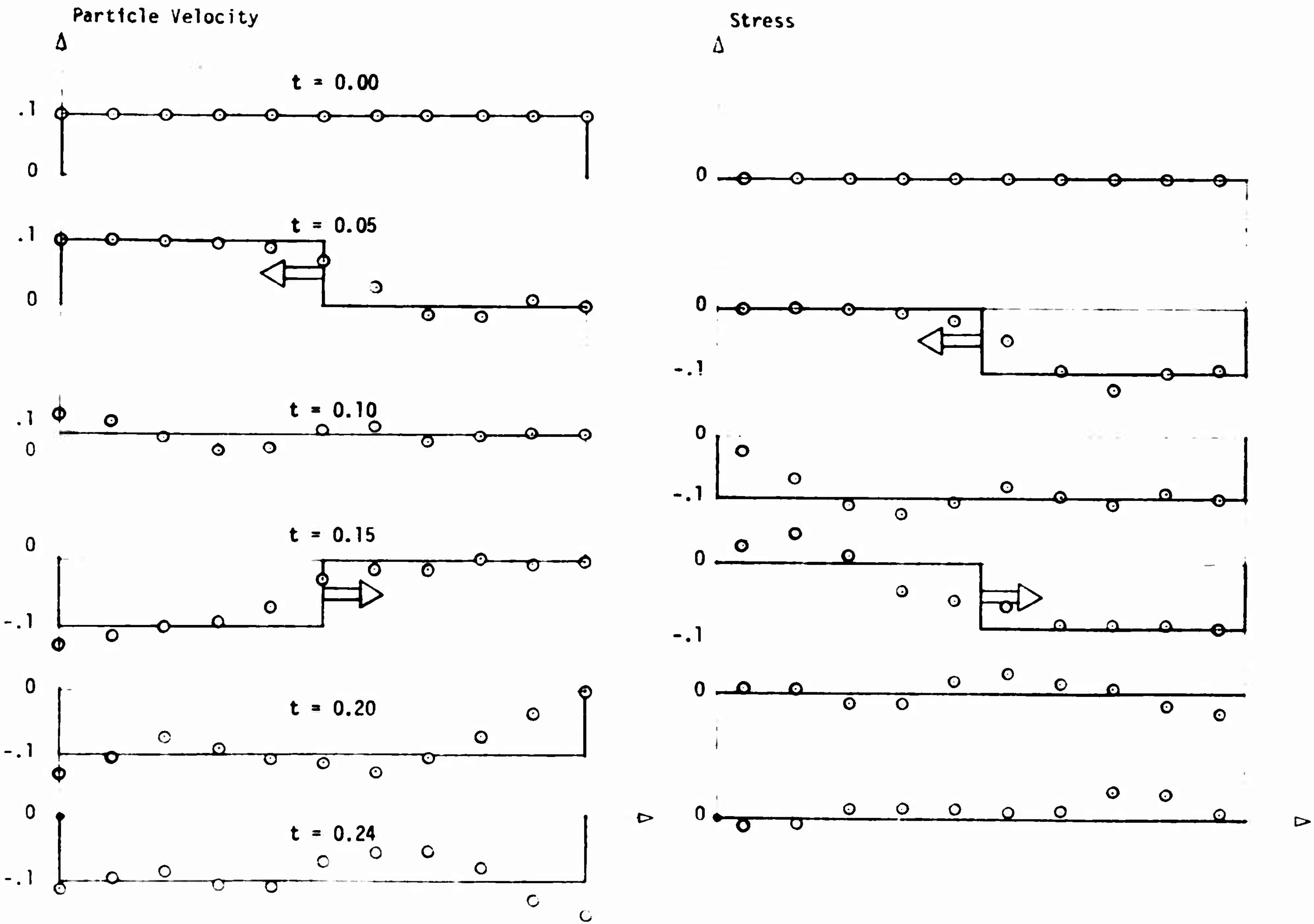


Figure 12

17. Closure and Recommendations for Future Work

In the preceding sections we have presented a theory for contact-impact problems together with the numerical development of a Hertzian contact-impact model. The computer program FEAP 74 has been modified to include the model and has successfully solved a contact problem and an impact problem. The work reported herein must be considered initiatory; the general theories and their numerical implementation have not been completed. The problem is of such a complicated nature and the literature existing prior to this study was so meager that we consider it fitting to document the work completed thus far.

We have attempted to qualify each stage of the development throughout the report, however, it may be fitting to reiterate future work which we consider to be essential for numerical models to be effective and efficient tools for predictive analyses.

(1) The restriction of Hertzian type contact must be removed. This involves the non-trivial task of finding appropriate numerical methods to handle the $\underset{\sim}{\chi}^{\alpha}$ maps.

(2) Improved methods for solving the set of nonlinear algebraic equations must be found. We have suggested two methods which should be considered: (a) Substructure the problem about the contact regime so that a more efficient forming and factoring of the tangent stiffness can be performed; and (b) Since the impact problem is a wave propagation problem an explicit time integration of the equations of motion should be explored. In complex situations the explicit integration method may have severe stability limitations which could make it unacceptable.

(3) Methods of utilizing the shock conditions need to be explored further. We have noted some peculiar anomalies when the bodies separate. These appear to be caused by a shock like separation phenomena.

(4) When the wave propagation property of the impact problem is ignored by taking time steps greatly in excess of the transit times in a body the computed response is meaningless. Under such situations the bodies rebound within a single time step. Currently the rebound velocity is much too large. When the shock conditions are used for a class of problems where the response desired is in the target instead of in the impactor, it may be expedient to take a large time step. Methods should be explored to accomplish this capability.

The above recommendations for future work should in no way minimize what has been accomplished by the present study. For the first time a contact-impact theory in the form of a variational problem has been presented in a general form. This formulation was motivated by the fact that numerical solutions would be obtained by a finite element method. In addition the necessary foundation for the numerical solution has been thought out and within this context a computer program has been developed for Hertzian contact-impact problems.

The implementations considered here have produced results which are hopeful signs for the eventual success of the more general impact problems.

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APPENDICES

A. Input Instructions for Contact/Impact Problems

In order to analyze contact/impact problems in FEAP, users must prepare the data for a time dependent analysis. This will include the following Data Type Identification Cards (see Section 1, Appendix B):

FEAP 74

MATERIAL

NODAL

ELEMENT

CONTACT (for impact problems only)

loadings

INITIAL CONDITIONS (if non-zero)

and

VISCOE (for quasi-static contact problems)

or

IMPLICIT (for impact problems)

In performing the necessary solution to (62) the full Newton-Raphson method must be employed; this is controlled by the data in col. 76-80 of the first card following the VISCOE or IMPLICIT card, and consists of a negative number (negative uses full Newton-Raphson iteration with the absolute value of the number giving the number of iterations to be performed before going to the next time step). As an example of the required input data Table A shows the input data used for the impact problem reported in Section 16.

```
14.19.14 OUTPUT
    FEAP74 * * ONE DIMENSIONAL BAR CONTACT PROBLEM
   1      X
   1      U
   3
  13 NODAL
   1                   0.
  11                   10.
  13          111111   10.
  11 ELEMENTS
   1    1            1   1
  1   2
  11      2
 11   12  13
   3 INITIAL CONDITIONS
    DISPLACEMENT
    VELOCITY
    ACCELERATIONS
   1    1              .1
  11    0              .1


   2 MATERIALS
   1 ELM09
  100.      1.         .01
   2 ELM05
   1
    CONTACT
   1        1.0        1.0E+30
  11    1

   1 IMPLICIT
  1.0       5   1   1  13   1  11   0   0  0.25                -3
  .01      25   1   1  13   1  11   0   0  0.25                -3
    STOP
  .01      10   1   1  13   9  11   0   0  0.25                -3
```

B. Input Instructions for FEAP 74

The input instructions for the description of a finite element mesh, together with the initial and boundary conditions, is described by subroutine MANUAL listed on the following pages. The input of the contact surface for impact problems is described in Table I of this report.

The description of material properties for the contact element is described in Section 14. For material properties for other elements in FEAP special input instructions must be supplied.

```
      SUBROUTINE MANUAL                                                    MAN   1C
C                                                                          MAN   2C
C**** USER INSTRUCTIONS AND INPUT FORMATS FOR FEAP-74  *****************   MAN   3C
C                                                                          MAN   4C
C                                                                          MAN   5C
C     FINITE ELEMENT ANALYSIS PROGRAM                                      MAN   6C
C                                                                          MAN   7C
C      ****** I N D E X ******                                             MAN   8C
C                                                                          MAN   9C
C  SECTION    WORD      C O N T E N T                                      MAN  10C
C                                                                          MAN  11C
C    1.0                DATA DIRECTORY WORDS                               MAN  12C
C    2.0      FEAP74    PROBLEM INITIATION AND CONTROL CARDS               MAN  13C
C    2.1      REMARK    REMARK CARD                                        MAN  14C
C    2.2      TITLE     TITLE CHANGE                                       MAN  15C
C    2.3      STOP      EXECUTION TERMINATION                              MAN  16C
C    3.0      MATERI    MATERIAL CHARACTERIZATION                          MAN  17C
C    4.0      NODAL     SEQUENTIAL NODAL GENERATION                        MAN  18C
C    4.1      GENERA    NON-SEQUENTIAL NODAL GENERATION                    MAN  19C
C    4.2      BOUNDA    BOUNDARY CODE INPUT                                MAN  20C
C    4.3      POLAR     POLAR TO CARTESIAN CONVERSION                      MAN  21C
C    5.0      ELEMEN    ELEMENT GENERATION                                 MAN  22C
C    5.1      BLOCK     BLOCK MESH GENERATION                              MAN  23C
C    6.0      VECTOR    USER VECTOR INPUT                                  MAN  24C
C    6.1      INITIA    INITIAL CONDITION                                  MAN  25C
C    7.0      FORCE     GENERALIZED NODAL FORCE                            MAN  26C
C    7.1      BLOADS    SURFACE LOADS                                      MAN  27C
C    7.2      ELOADS    ELEMENT LOADS                                      MAN  28C
C    7.3                PROPORTIONAL LOADS                                 MAN  29C
C    8.0      SOLVE     INITIATION OF STATIC SOLUTION                      MAN  30C
C             RESOLV    STATIC SOLUTION                                    MAN  31C
C    8.1      EXPLIC    EXPLICIT DYNAMIC INTEGRATION                       MAN  32C
C    8.2      IMPLIC    IMPLICIT INTEGRATION                               MAN  33C
C             VISCOE    IMPLICIT TIME INTEGRATION                          MAN  34C
C    8.3      MESH      MESH CHECK                                         MAN  34C
C             PLOT      PLOT MESH                                          MAN  34C
C    8.4      FOURIE    FOURIER SERIES HARMONICS                           MAN  35C
C    9.0                OUTPUT CONTROL                                     MAN  35C
C                                                                          MAN  36C
C        FEAP74 IS A GENERAL (F)INITE (E)LEMENT (A)NALYSIS (P)ROGRAM       MAN  37C
C        WHICH FURNISHES TO THE USER MESH INPUT/OUTPUT, ELEMENT ASSEMBLY   MAN  38C
C        AND SOLUTION OF EQUATIONS (LINEAR, IMPLICIT AND EXPLICIT TIME     MAN  39C
C        DEPENDENT, NONLINEAR), PRESCRIBED GENERALIZED NODAL FORCES,       MAN  40C
C        PRESCRIBED NODAL AND ELEMENT DATA, AND OUTPUT OF THE              MAN  41C
C        GENERALIZED DISPLACEMENTS AND FORCES.  ELEMENT MATRICES FOR TWO   MAN  42C
C        AND THREE DIMENSIONAL LINEAR ELASTICITY, SHELLS, PLATES, AND      MAN  43C
C        FIELD (LAPLACE EQUATION) PROBLEMS ARE AVAILABLE.  ALTERNATIVELY   MAN  44C
C        USERS MAY SUPPLY THEIR OWN ELEMENT LIBRARY BY PROVIDING A         MAN  45C
C        SUBROUTINE CALLED ELMTNN.  WHERE NN IS A TWO DIGIT NUMBER         MAN  46C
C        (01-10), IDENTIFYING THE ELEMENT SUBROUTINE. EACH ELEMENT         MAN  47C
C        SUBROUTINE HAS AT LEAST FOUR BASIC FUNCTIONS WHICH ARE            MAN  48C
C        DELINEATED BY A SWITCHING PARAMETER, ISW. IN THE SUBROUTINE.      MAN  49C
C                                                                          MAN  50C
```

```
C        ELMTNN(N,MA,NDIM,NDF,NEL,NEL1,NSTF,NSIZV,NVEC,MCT,DM,D,XYZ,
C        IX,F,FORCE,ESTIF,U,VECT,ISW)
C
C                       N IS ELEMENT NUMBER.
C                       MA IS THE MATERIAL NUMBER.
C                       NDIM IS SPATIAL DIMENSION. 1,2,OR 3.
C                       NDF IS NUMBER OF DEGREES OF FREEDOM PER NODE.
C                       NEL IS THE NUMBER OF EXTERNAL NODES PER ELEMENT
C                       NEL1 IS DIMENSION OF ELEMENT PROPERTY ARRAY.
C                       NSTF IS THE SIZE OF THE ELEMENT STIFFNESS.
C                       NSIZV IS THE SIZE OF UTILITY VECTORS.
C                       NVEC IS THE NUMBER OF UTILITY VECTORS.
C                       MCT IS A PRINTER LINE COUNTER.
C                       DM IS A PARAMETER FOR MATERIAL IDENTIFICATION.
C                       D(L,1) IS MATERIAL PROPERTY MATRIX (63 CELLS).
C                       XYZ(NDIM,1) ARE NODAL COORDINATES.
C                       IX(NEL1,1) ARE ELEMENT  PROPERTIES, NODES, ETC.
C                       F(NDF,1) ARE NODAL GENERALIZED FORCES.
C                       FORCE(NSTF,2) IS ELEMENT FORCE VECTOR TO BE
C                       COMPUTED. COLUMN 2 IS LUMPED MASS
C                       ESTIF(NSTF,NSTF) IS ELEMENT MATRIX TO BE
C                       COMPUTED.
C                       VECT(NSIZV,1) ARE PRESCRIBED NODAL OR ELEMENT
C                       QUANTITIES, TEMPERATURES ETC.
C                       U(NDF,1) IS SOLUTION VECTOR.
C                       ISW IS SWITCHING PARAMETER.
C
C                          ISW=1. ** MATERIAL CHARACTERIZATION**
C                          ISW=2. ** CHECK ELEMENT FOR POSITIVE AREA *
C                          ISW=3. ** ELEMENT STIFFNESS AND
C                                    LOAD COMPUTATION**
C                          ISW=4. ** ELEMENT STRESSES AND PRINTOUT**
C                          ISW=5. ** ELEMENT LOAD COMPUTATION ONLY**
C                          ISW=6. ** NONLINEAR GENERALIZED FORCES**
C                          OTHER ISW MAY BE USED FOR SPECIAL PURPOSES.
C
C        USERS CAN GENERATE SURFACE LOADINGS BY PROVIDING SLDNN
C        SUBROUTINES (WHERE NN IS A TWO DIGIT NUMBER BETWEEN 01 AND 05)
C        THAT SPECIFY THE LOAD ROUTINE.  THE SUBROUTINE IS ACCESSED BY
C        THE CALL TO
C
C        SLDNN(NDIM,NDF,NDP,NPRES,IPRES,PP,XYZ,FS)
C
C                       WHERE IN ADDITION TO QUANTITIES DEFINED ABOVE
C                       FOR ELMTNN.
C                       NDP IS THE DIMENSION OF LOADED SURFACE
C                       NPRES IS NUMBER OF LOADED NODES (MAX 8)
C                       IPRES(8) ARE NODE NUMBERS OF LOADED NODES.
C                       PP(8) ARE LOAD VALUES AT CORRESPONDING IPRES
C                           NODES.
C                       FS(6,8) ARE THE COMPUTED GENERALIZED (NODAL)
C                           FORCES FOR EACH DEGREE OF FREEDOM AT EACH
C                           IPRES NODE.
```

```
C        SEE SECTION 7.1 FOR DATA INPUT DETAILS.                                  MAN104C
C                                                                                 MAN105C
C                                                                                 MAN106C
C        INTEGRATION TABLE IS ACCESSED BY THE CALL                                MAN107C
C                                                                                 MAN108C
C          CALL  INTEGL(LIM.NC1.NDIM.LINT.STUW)                                   MAN109C
C                                                                                 MAN110C
C                          STUW(4.M)  INTEGRATION POINTS AND WEIGHTS.             MAN111C
C                          **NOTE** M MUST BE SET EXPLICITLY AND BE LARGER        MAN112C
C                          THAN OR EQUAL TO LINT.                                 MAN113C
C                          LINT - RETURNS WITH NUMBER INTEGRATION POINTS.         MAN114C
C                          NC1 = 0  RETURNS GAUSS POINTS AND WEIGHTS IN           MAN115C
C                               STUW.                                             MAN116C
C                               LIM = 1 TO 5 IS NUMBER OF GAUSS POINTS/DIR-       MAN117C
C                               ECTION.                                           MAN118C
C                          NC1 = 1  RETURNS A SPECIAL 3-D GAUSS FORMULA.          MAN119C
C                               SET LIM = 1 FOR 6 PT. CUBIC ACCURACY              MAN120C
C                               SET LIM = 2 FOR 14 PT. QUINTIC ACCURACY.          MAN121C
C                          NC1 = 2  RETURNS TRIANGULAR INTEGRATION FORMULA        MAN122C
C                               SET LIM = 1 FOR 1 PT. LINEAR ACCURACY.            MAN123C
C                               SET LIM = 2 FOR 3 PT. QUADRATIC ACCURACY.         MAN124C
C                               SET LIM = 3 FOR 7 PT. QUARTIC ACCURACY.           MAN125C
C                                                                                 MAN126C
C                                                                                 MAN127C
C      1.) DATA TYPE IDENTIFICATION CARDS (I5.1X.12A6).                           MAN128C
C                                                                                 MAN129C
C        EACH DATA SEGMENT IS PRECEDED BY A CARD WHICH IDENTIFIES THE             MAN130C
C        TYPE OF DATA AND LIMITS ON THE AMOUNT OF DATA WHICH IMMEDIATELY          MAN131C
C        FOLLOWS THE CARD.  EXCEPT AS NOTED THE DATA SEGMENTS MAY APPEAR          MAN132C
C        IN ANY ORDER.  THE IDENTITY CARDS MAY ALSO AID THE USER IN               MAN133C
C        INTERPRETTING THE INPUT DATA CARDS.  AS SUPPLIED THERE ARE               MAN134C
C        TWENTY-FIVE DIFFERENT DATA IDENTIFICATION CARDS.  THESE ARE              MAN135C
C                                                                                 MAN136C
C        COL  7 TO 12      IDENTITY(RESTRICTIONS)                                 MAN137C
C                                                                                 MAN138C
C        FEAP74            START OF EACH PROBLEM (MUST PRECEDE ALL OTHER          MAN139C
C                          DATA).                                                 MAN140C
C        TITLE             CHANGE OUTPUT PAGE HEADINGS                            MAN141C
C        REMARK            COMMENTS ON OUTPUT                                     MAN142C
C        MATERI            MATERIAL CHARACTERIZATION.                             MAN143C
C        NODAL             NODAL CARDS                                            MAN144C
C        POLAR             POLAR CONVERSION. (PRECEDE BY NODAL.GENERA. OR         MAN145C
C                          BLOCK )                                                MAN146C
C        ELEMEN            ELEMENT CONNECTION CARDS.                              MAN147C
C        GENERA            GENERATE NODES IN A LINEAR PATH BY ANY INCREMNT        MAN148C
C        BLOCK             GENERATE ALL MESH DATA (BOTH NODAL AND ELEMENT)        MAN149C
C                          FOR A 2.(OR 3)DIMENSIONAL REGION WHOSE BOUNDARY        MAN150C
C                          MAY BE DEFINED BY 4(8)OR 8(20)COLLOCATED POINTS        MAN151C
C        BOUNDA            BOUNDARY CODE PRESCRIPTION (PRECEDE BY NODAL OR        MAN152C
C                          GENERATE OR BLOCK)                                     MAN153C
C        VECTOR            PRESCRIBED NODAL OR ELEMENT DATA (PRECEDE BY           MAN154C
C                          NODAL. OR GENERA AND ELEMEN. OR BLOCK)                 MAN155C
C        FORCE             NODAL GENERALIZED FORCES (PRECEDE BY NODAL OR          MAN156C
C                          GENERA OR BLOCK).                                      MAN157C
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C       BLOADS          SURFACE LOADINGS (SAME AS FORCE).                     MAN158C
C       ELOADS          ELEMENT LOADINGS (SAME AS FORCE).                     MAN159C
C       MESH            CHECK CONSISTENCY OF MESH ONLY (SAME AS SOLVE)        MAN160C
C       PLOT            PLOT MESH (SAME AS SOLVE)                             MAN161C
C       INITIA          INITIAL CONDITION PRESCRIPTION FOR DYNAMIC            MAN162C
C                       ANALYSIS (PRECEDE BY NODAL. GENERA OR BLOCK)          MAN163C
C       SOLVE           COMPLETE FORMULATION AND SOLUTION FROM ELEMENTS       MAN164C
C                       (PRECEDE BY MATERI.NODAL OF GENERA. AND ELEMEN        MAN165C
C                       OR PRECEDE BY MATERI AND BLOCK )                      MAN166C
C       RESOLV          USE PREVIOUS PROBLEM DESCRIPTION WITH NEW LOAD        MAN167C
C                       ONLY (PRECEDE BY SOLVE AND NEW LOADING CARDS).        MAN168C
C       EXPLIC          DYNAMIC SOLUTION BY EXPLICIT INTEGRATION. (SAME       MAN169C
C                       AS SOLVE)                                             MAN170C
C       IMPLIC          IMPLICIT INTEGRATION OF DYNAMIC PROBLEMS              MAN171C
C                       (PRECEDE BY SAME DATA AS FOR SOLVE)                   MAN172C
C       VISCOE          QUASI-STATIC LINEAR VISCOELASTIC INTEGRATION          MAN173C
C                       (PRECEDE BY SAME DATA AS FOR SOLVE)                   MAN174C
C       FOURIE          FOURIER COMPOSITION (SAME AS SOLVE)                   MAN175C
C       ADDUP           ACCUMULATE FOURIER SOLUTION (AFTER FOURIE)            MAN176C
C       STOP            NORMAL EXIT (MUST FOLLOW ALL DATA)                    MAN177C
C                                                                             MAN178C
C       ***NOTE*** EACH IDENTIFIER IS PUNCHED STARTING IN COL 7 (LEFT         MAN179C
C           JUSTIFIED).                                                       MAN180C
C                                                                             MAN181C
C     EXCESS CARDS MAY EXIST BETWEEN EACH SECTION OF DATA. HOWEVER.           MAN182C
C     THE DATA TO BE USED MUST IMMEDIATELY FOLLOW THE TYPE CARD AND           MAN183C
C     MUST BE IN PROPER ORDER.  NO PARTICULAR ORDER OF THE TYPE               MAN184C
C     CARDS IS NECESSARY EXCEPT THAT THE FEAP74 CARD MUST ALWAYS BE           MAN185C
C     THE FIRST CARD IN EACH SET OF DATA. AND RESTRICTIONS MUST BE            MAN186C
C     OBSERVED.                                                               MAN187C
C                                                                             MAN188C
C                                                                             MAN189C
C    2.) PROBLEM INITIATION AND CONTROL CARDS                                 MAN190C
C                                                                             MAN191C
C       CARD 1. (6X.12A6)                                                     MAN192C
C                                                                             MAN193C
C       COL  7 TO 12    MUST CONTAIN WORD FEAP74                              MAN194C
C       COL 13 TO 78    OUTPUT PAGE HEADER                                    MAN195C
C                                                                             MAN196C
C       CARD 2. (I5.1X.3A6)                                                   MAN197C
C                                                                             MAN198C
C       COL  1 TO 5     NDIM - SPATIAL DIMENSION OF PROBLEM (1 TO 3)          MAN199C
C       COL  7 TO 12    NAMES TO BE PRINTED AS OUTPUT HEADERS TO              MAN200C
C       COL 13 TO 18    COORDINATES - IF BLANK SET TO 1.2.3 AS NEEDED.        MAN201C
C       COL 19 TO 24                                                          MAN202C
C                                                                             MAN203C
C       CARD 3. (I5.1X.6A6)                                                   MAN204C
C                                                                             MAN205C
C       COL  1 TO  5    NDF - NUMBER OF UNKNOWNS PER NODE (1 TO 6)            MAN206C
C       COL  7 TO 12    NAMES TO BE PRINTED AS OUTPUT HEADERS OF THE          MAN207C
C       COL 13 TO 18    GENERALIZED DISPLACEMENTS AND FORCES - IF             MAN208C
C           ........    BLANK SET TO 1.2.3.4.5.6 AS NECESSARY                 MAN209C
C       COL 37 TO 42                                                          MAN210C
C                                                                             MAN211C
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C        CARD 4. (6I5,5F10.0)                                                   MAN2120
C                                                                               MAN2130
C        COL  1 TO 5      NEN - MAXIMUM NUMBER OF NODES CONNECTED TO ANY        MAN2140
C                         ELEMENT (1 TO 20).                                    MAN2150
C        COL  6 TO 10     NEXTRA - INCREASES ELEMENT MATRIX SIZE FROM           MAN2160
C                         NDF*NEN TO NDF*NEN + NEXTRA                           MAN2170
C        COL 11 TO 15     IREC - COMPUTE GENERALIZED FORCE CHECK IF             MAN2180
C                         NONZERO (FOR TIME INVARIANT ANALYSIS ONLY)            MAN2190
C        COL 16 TO 20     MBAN - MAXIMUM EXPECTED BANDWIDTH. DEFAULT IS         MAN2200
C                         SET TO 100. USED AS AN ERROR CHECK TO PREVENT         MAN2210
C                         RUNNING WITH AN OBVIOUS ERROR.                        MAN2220
C        COL 21 TO 25     IBUF - BUFFER SIZE FOR STORAGE OF HISTORY             MAN2230
C                         EFFECTS IN TIME DEPENDENT ANALYSIS. DEFAULT IS        MAN2240
C                         IBUF = ISZDT/20                                       MAN2250
C        COL 26 TO 30     NC1  - USER INTEGER CONSTANT                          MAN2260
C        COL 31 TO 40     CON1 - USER DEFINED CONSTANT                          MAN2270
C        COL 41 TO 50     CON2 - USER DEFINED CONSTANT                          MAN2280
C        COL 51 TO 60     CON3(1) - USER DEFINED CONSTANT                       MAN2290
C        COL 61 TO 70     CON3(2) - USER DEFINED CONSTANT                       MAN2300
C        COL 71 TO 80     CON3(3) - USER DEFINED CONSTANT                       MAN2310
C                                                                               MAN2320
C      2.1) *REMARK* USER COMMENTS ON OUTPUT. (6X,12A6)                         MAN2330
C                                                                               MAN2340
C        SUBSEQUENT CARDS                                                       MAN2350
C                                                                               MAN2360
C        COL  7 TO 12     MUST CONTAIN REMARK                                   MAN2370
C        COL 13 TO 78     STATEMENTS TO BE OUTPUT , USE AS MANY REMARK          MAN2380
C                         CARDS AS DESIRED. INSERT BEFORE ANY TYPE CARD.        MAN2390
C                                                                               MAN2400
C      2.2) TITLE CHANGE ON OUTPUT (6X,12A6)                                    MAN2410
C                                                                               MAN2420
C        COL  7 TO 12     MUST CONTAIN TITLE                                    MAN2430
C        COL 13 TO 78     NEW TITLE DESCRIPTOR                                  MAN2440
C                                                                               MAN2450
C      2.3) EXECUTION TERMINATION (6X,A4)                                       MAN2460
C                                                                               MAN2470
C        COL  7 TO 10     MUST CONTAIN STOP. INSERT AFTER LAST PROBLEM.         MAN2480
C                                                                               MAN2490
C                                                                               MAN2500
C     3.) MATERIAL CHARACTERIZATION (I5,1X,12A6)                                MAN2510
C                                                                               MAN2520
C        COL  1 TO  5     NUMMAT - NUMBER OF DIFFERENT MATERIAL CHARACT-        MAN2530
C                         ERIZATIONS TO FOLLOW.                                 MAN2540
C        COL  7 TO 12     MUST CONTAIN WORD MATERI                              MAN2550
C                                                                               MAN2560
C        THE FOLLOWING CARDS ARE SUPPLIED FOR EACH MATERIAL TO BE CHARAC        MAN2570
C        TERIZED (MUST BE EXACTLY NUMMAT SETS OF CARDS)                         MAN2580
C                                                                               MAN2590
C        CARD 1.  ELEMENT SELECTOR CARD (I5,1X,A5,11A6)                         MAN2600
C                                                                               MAN2610
C        COL  1 TO  5     MATERIAL NUMBER (1 TO NUMMAT                          MAN2620
C        COL  7 TO 11     ELMNN - WHERE NN IS NUMBER OF ELEMENT CLASS (01       MAN2630
C                         TO 10) TO WHICH THE CHARACTERIZATION BELONGS.         MAN2640
C        COL [illegible]  ALPHANUMERIC INFORMATION TO BE OUTPUT.                MAN2650
```

```
C
C        CARD 2.1. ETC. ** USER DEFINED FOR EACH ELEMENT TYPE PROVIDED.
C
C        EXCESS BLANK CARDS ARE PERMISSIBLE BETWEEN EACH MATERIAL SET.
C
C     4.) NODAL CARDS (I5.1X.A6)
C
C        COL  1 TO  5    NUMNP - NUMBER OF NODAL POINTS
C        COL  7 TO 12    MUST CONTAIN NODAL
C
C        SUBSEQUENT CARDS  LAST NODAL CARD MUST NOT BE GENERATED.
C        (I5.I15.3F10.0)
C
C        COL  1 TO  5    NODE NUMBER
C        COL 15          1 IF 1 DISPLACEMENT IS SPECIFIED
C        COL 16          1 IF 2 DISPLACEMENT IS SPECIFIED
C        COL 17          1 IF 3 DISPLACEMENT IS SPECIFIED
C        COL 18          1 IF 4 DISPLACEMENT IS SPECIFIED
C        COL 19          1 IF 5 DISPLACEMENT IS SPECIFIED
C        COL 20          1 IF 6 DISPLACEMENT IS SPECIFIED
C        COL 21 TO 30    1 COORDINATE VALUE
C        COL 31 TO 40    2 COORDINATE VALUE * AS REQUIRED
C        COL 41 TO 50    3 COORDINATE VALUE
C
C     NODAL CARDS MUST BE IN ORDER. MISSING NODES ARE INTERPOLATED
C     LINEARLY FROM INPUT NODES. IF SUCCEEDING CARDS HAVE IDENTICAL
C     BOUNDARY CODES. THIS BOUNDARY CODE WILL BE ASSIGNED TO THE INTER-
C     VENING NODES. IN ALL OTHER CASES THE BOUNDARY CODE IS SET TO ZERO
C     *TERMINATE ON NODE NUMNP OR A BLANK CARD*
C
C     4.1) NON SEQUENTIAL NODAL GENERATOR OPTION. (I5.1X.A6)
C
C        COL  1 TO  5    NUMBER OF NODAL POINTS
C        COL  7 TO 12    MUST CONTAIN GENERA
C
C        SUBSEQUENT CARDS (2I5.I10.3F10.0)
C
C        COL  1 TO  5    NODE-NUMBER
C        COL  6 TO 10    NODE-NUMBER-INCREMENT WHICH WILL BE SUCESSIVELY
C                        ADDED TO NODE-NUMBER UNTIL SUM IS GREATER THAN
C                        NODE-NUMBER ON FOLLOWING CARD (ALGEBRAIC).
C        COL 15 TO 20    BOUNDARY CODE. SAME AS INPUT FOR NODAL.
C                        IF SUCCEEDING CARDS HAVE IDENTICAL BOUNDARY
C                        CODES. THIS BOUNDARY CODE WILL BE ASSIGNED TO
C                        THE INTERVENING NODES. IN ALL OTHER CASES THE
C                        BOUNDARY CODE IS SET TO ZERO.
C        COL 21 TO 30    1 COORDINATE VALUE  *
C        COL 31 TO 40    2 COORDINATE VALUE  * AS REQUIRED *
C        COL 41 TO 50    3 COORDINATE VALUE  *
C
C        *TERMINATE WITH BLANK CARD *
C
C     4.2 BOUNDARY CODE PATCH-UP OPTION. (6X.A6)
C
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C         COL   7 TO 12     MUST CONTAIN BOUNDA                                            MAN318C
C                                                                                          MAN319C
C         SUBSEQUENT CARDS. (8I5)                                                          MAN320C
C                                                                                          MAN321C
C         COL   1 TO  5     N. NODE NUMBER TO HAVE REDEFINED BOUNDARY CODE.                MAN322C
C         COL   6 TO 10     NX. GENERATOR INCREMENT TO BE ADDED ALGEBRAI-                  MAN323C
C                           CALLY TO N. UNTIL SUM EXCEEDS (MAX OR MIN) THE                 MAN324C
C                           N OF THE FOLLOWING CARD.                                       MAN325C
C         COL  11 TO 15     IBC(I). (I=1.2...NDF) CODE FOR SPECIFYING FORCE                MAN326C
C         COL  16 TO 20     OR DISPLACEMENT BOUNDARY CONDITIONS.                           MAN327C
C         COL   ...                                                                        MAN328C
C                                IBC(I) .EQ. 0. FORCE SPECIFIED.                           MAN329C
C                                IBC(I) .GT. 0. DISPLACEMENT SPECIFIED. NO                 MAN330C
C                                INTERVENING GENERATION.                                   MAN331C
C                                IBC(I) .LT. 0. DISPLACEMENT SPECIFIED.                    MAN332C
C                                GENERATE BETWEEN MISSING NODES IN ALGEBRAIC               MAN333C
C                                INCREMENTS OF NX.                                         MAN334C
C                                                                                          MAN335C
C         * TERMINATE WITH A BLANK CARD. *                                                 MAN336C
C                                                                                          MAN337C
C       4.3) POLAR OR CYLINDRICAL COORDINATE CONVERSION TO CARTESIAN                       MAN338C
C            COORDINATES (6X.A6)                                                           MAN339C
C                                                                                          MAN340C
C         COL   7 TO 12     MUST CONTAIN POLAR     (LEFT JUSTIFIED)                        MAN341C
C                                                                                          MAN342C
C         CARD 1.   (3I5.5X.2F10.0)                                                        MAN343C
C                                                                                          MAN344C
C         COL   1 TO  5     N1. FIRST NODE TO BE CONVERTED                                 MAN345C
C         COL   6 TO 10     N2. LAST  NODE TO BE CONVERTED                                 MAN346C
C         COL  11 TO 15     N3. INCREMENT ADDED (ALGEBRAICALLY). N1 TO N2                  MAN347C
C         COL  21 TO 30     X0. ORIGIN OF POLAR X-COORDINATE                               MAN348C
C         COL  31 TO 40     Y0. ORIGIN OF POLAR Y-COORDINATE                               MAN349C
C                                                                                          MAN350C
C      5.) ELEMENT CARDS (I5.1X.A6)                                                        MAN351C
C                                                                                          MAN352C
C         COL   1 TO  5     NUMEL - NUMBER OF ELEMENTS                                     MAN353C
C         COL   7 TO 12     MUST CONTAIN ELEMEN                                            MAN354C
C                                                                                          MAN355C
C         SUBSEQUENT CARDS (4I5.20I3/20I4)                                                 MAN356C
C                                                                                          MAN357C
C         CARD 1.                                                                          MAN358C
C                                                                                          MAN359C
C         COL   1 TO  5     ELEMENT NUMBER                                                 MAN360C
C         COL   6 TO 10     MATERIAL NUMBER                                                MAN361C
C         COL  11 TO 15     NUMBER OF SUBSEQUENT ELEMENTS USING SAME                       MAN362C
C                           STIFFNESS MATRIX * SAVES RECOMPUTATION OF                      MAN363C
C                           SIMILAR MATRICES.  ELEMENT MUST ALSO HAVE                      MAN364C
C                           SAME ELEMENT FORCE VECTOR * IF THESE ARE                       MAN365C
C                           IN THE STIFFNESS SUBROUTINE *                                  MAN366C
C         COL  16 TO 20     PRINT ELEMENT MATRIX IF NONZERO.                               MAN367C
C         COL  21 TO 23     IXD(1) ELEMENT INCREMENT ARRAY ON NODE 1.                      MAN368C
C         COL  24 TO 26     IXD(2)   * IF NOT INPUT IS SET AUTOMATICALLY                   MAN369C
C                           UP TO                                                          MAN370C
C         COL [illegible]   I [illegible]    FOR SERENDIPITY ELEMENTS * SEE REPORT         MAN371C
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C                                                                          MAN372C
C        CARD 2.                                                           MAN373C
C                                                                          MAN374C
C        COL  1 TO  4     NODE 1                                           MAN375C
C        COL  5 TO  8     NODE 2                                           MAN376C
C        COL  9 TO 12     NODE 3          IX(NEL1,1) ARRAY                 MAN377C
C                             CONTINUE IN I4 FORMAT TO A MAXIMUM           MAN378C
C        COL 77 TO 80     NODE 20                                          MAN379C
C                                                                          MAN380C
C        ELEMENT CARDS MUST BE IN ORDER. MISSING ELEMENTS ARE GENERATED    MAN382C
C        BY INCREMENTING NODES.                                            MAN383C
C        LAST ELEMENT CARD MUST NOT BE GENERATED.                          MAN384C
C        * TERMINATE ON ELEMENT NUMEL OR A BLANK CARD *                    MAN385C
C                                                                          MAN386C
C      5.1) BLOCK GENERATOR. GENERATES ALL MESH DATA.(6X,A6)               MAN387C
C                                                                          MAN388C
C        COL  1 TO  5     NUMBER OF NODAL POINTS TO BE GENERATED.          MAN389C
C        COL  7 TO 12     MUST CONTAIN BLOCK                               MAN390C
C                                                                          MAN391C
C        SUBSEQUENT CARDS (10I5/6(4X,I6)/(10X,3F10.0))                     MAN392C
C                                                                          MAN393C
C        CARD 1.                                                           MAN394C
C                                                                          MAN395C
C        COL  1 TO  5     NN. NUMBER OF POINTS REQUIRED TO DEFINE          MAN396C
C                         BOUNDARY OF REGION. FOR 2-DIM., NN=4 OR 8.       MAN397C
C                         FOR 3-DIM., NN= 8 OR 20.                         MAN398C
C        COL  6 TO 10     NUMBER OF ELEMENTS IN R-DIRECTION.               MAN399C
C        COL 11 TO 15     NUMBER OF ELEMENTS IN S-DIRECTION.               MAN400C
C        COL 16 TO 20     NUMBER OF ELEMENTS IN T-DIRECTION.               MAN401C
C        COL 21 TO 25     INITIAL NODE NUMBER. DEFAULT = 1.                MAN402C
C        COL 26 TO 30     INITIAL ELEMENT NUMBER. DEFAULT = 1.             MAN403C
C        COL 31 TO 35     MATERIAL NUMBER OVER REGION. DEFAULT = 1         MAN404C
C        COL 36 TO 40     BOUNDARY CODE SKIP. A NON-ZERO ENTRY WILL OMIT   MAN405C
C                         SETTING ALL INTERIOR BOUNDARY CODES TO ZERO.     MAN406C
C        COL 41 TO 45     IREUSE - REUSE ELEMENT STIFFNESS OPTION - USES   MAN407C
C                         EACH ELEMENT STIFFNESS IREUSE TIMES BEFORE       MAN408C
C                         GENERATING A NEW ELEMENT STIFFNESS MATRIX.       MAN409C
C        COL 46 TO 50     ELEMENT STIFFNESS-PRINT. A NON-ZERO ENTRY  WILL  MAN410C
C                         CAUSE PRINT-OUT OF FIRST ELEMENT.                MAN411C
C        COL 51 TO 55     IMSH - IF NONZERO PRINT COMPUTED NODES.          MAN412C
C        COL 56 TO 60     IELM - IF NONZERO PRINT COMPUTED ELEMENTS.       MAN413C
C                                                                          MAN414C
C        CARD 2.  (BOUNDARY CODE AS DEFINED IN NODAL CARD.)                MAN415C
C                                                                          MAN416C
C        COL  5 TO 10     BOUNDARY CODE OVER FACE -R.                      MAN417C
C        COL 15 TO 20     BOUNDARY CODE OVER FACE +R.                      MAN418C
C        COL 25 TO 30     BOUNDARY CODE OVER FACE -S.                      MAN419C
C        COL 35 TO 40     BOUNDARY CODE OVER FACE +S.                      MAN420C
C        COL 45 TO 50     BOUNDARY CODE OVER FACE -T.                      MAN421C
C        COL 55 TO 60     BOUNDARY CODE OVER FACE +T.                      MAN422C
C                                                                          MAN423C
C        CARD 3.    REPEAT NN TIMES.)                                      MAN424C
C                                                                          MAN425C
C                           COORDINATE OF BOUNDARY-DEFINING POINT.         MAN426C
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C         COL 21 TO 30     2-COORDINATE OF BOUNDARY-DEFINING-POINT.            MAN427C
C         COL 31 TO 40     3-COORDINATE OF BOUNDARY-DEFINING-POINT.            MAN428C
C                                                                              MAN429C
C         NOTES. 1. BLOCK GENERATES ONLY 4 PT. QUADRILATERALS OR 8 PT.         MAN430C
C                   BRICKS.                                                    MAN431C
C                2. INPUT OF CARDS 3.) FOLLOW ORDER RULES FOR ELEMENT          MAN432C
C                   INPUT (SEE SECTION 5.)).                                   MAN433C
C                3. R-S-T ARE LOCAL COORDINATES.                               MAN433C
C                   I.E. (-1 .LE. R.S.T .LE. 1). WHERE R IS DIRECTED           MAN433C
C                   FROM NODE 1 TO 2. S IS IN PLANE OF FIRST THREE NODES       MAN433C
C                   AND T IS NORMAL TO R-S PLANE.                              MAN433C
C                                                                              MAN435C
C      6.)  VECTOR CARDS. I.E. USER DEFINED INPUT (I5.1X.A6)                   MAN436C
C                                                                              MAN437C
C         COL  1 TO  5     NVEC. NUMBER OF DIFFERENT VECTORS (7 MAX)           MAN438C
C         COL  7 TO 12     MUST CONTAIN VECTOR                                 MAN439C
C                                                                              MAN440C
C         SUBSEQUENT CARDS                                                     MAN441C
C                                                                              MAN442C
C         CARD 1.  (2I5)                                                       MAN443C
C                                                                              MAN444C
C         COL  1 TO  5     NSIZV. VECTOR LENGTH.COMMON TO ALL NVEC VECTORS     MAN445C
C         COL  6 TO 10     IPICK. CODED PARAMETER.                             MAN446C
C                                                                              MAN447C
C                          IPICK = 0. VECTORS ASSOCIATED WITH NODES            MAN448C
C                          IPICK = 1. VECTORS ASSOCIATED WITH DEG.FREEDOM      MAN449C
C                          IPICK = 2. VECTORS ASSOCIATED WITH ELEMENTS         MAN450C
C                                                                              MAN451C
C         CARD 2.  (6X. 2A6) REPEAT NVEC TIMES                                 MAN452C
C                                                                              MAN453C
C         COL  7 TO 18      DESCRIPTIVE TITLE FOR VECTOR                       MAN454C
C                                                                              MAN455C
C         CARD 3.  (2I5.7F10.0)                                                MAN456C
C                                                                              MAN457C
C         COL  1 TO  5     POSITION NUMBER OF VECTOR ELEMENT. 1 TO NSIZV       MAN458C
C         COL  6 TO 10     GENERATOR INCREMENT                                 MAN459C
C         COL 11 TO 20     VECTOR ELEMENT VALUE OF VECTOR 1                    MAN460C
C         COL 21 TO 30     VECTOR ELEMENT VALUE OF VECTOR 2                    MAN461C
C         COL  .......     AS REQUIRED FOR NVEC VECTORS                        MAN462C
C                                                                              MAN463C
C         LINEAR INTERPOLATION IS PERFORMED ON ALL VECTORS BETWEEN             MAN464C
C         NON-CONSECUTIVE POSITION NUMBERS SPECIFIED IN COL 1 TO 5             MAN465C
C         IF INCREMENT IS NONZERO.                                             MAN466C
C         IF DESCRIPTIVE TITLES OF ALL VECTORS ARE BLANK CARDS.                MAN467C
C         PRINTING OF THE VECTOR VALUES IS SUPPRESSED.                         MAN468C
C                                                                              MAN469C
C         * TERMINATE ON BLANK CARD *                                          MAN470C
C                                                                              MAN471C
C      6.)  INITIAL CONDITIONS FOR TIME DEPENDENT ANALYSIS.                    MAN472C
C                                                                              MAN473C
C         COL  1 TO  5     NINIT. NUMBER OF INITIAL CONDITION VECTORS          MAN474C
C         COL  7 TO 12     MUST CONTAIN INITIA                                 MAN475C
C                                                                              MAN476C
C         [illegible]                                                          MAN477C
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C
C       CG      (6X.2A6) REPEAT NICD TIMES
C
C       COL  7 TO 19     DESCRIPTIVE TITLE FOR INITIAL CONDITIONS
C
C       CARD 2.  (2I5.7F10.0)
C
C       COL  1 TO  5     POSITION NUMBER. AS IN VECTOR CARDS FOR IPICK=1
C       COL  6 TO 10     GENERATOR INCREMENT
C       COL 11 TO 20     INITIAL CONDITION 1
C       COL 21 TO 30     INITIAL CONDITION 2
C       COL ........     AS REQUIRED FOR NICD INITIAL CONDITIONS
C
C       INTERPOLATION BETWEEN INPUT VALUES AS DESCRIBED IN VECTOR INPUT
C       **** NOTE **** IF MISSING THE INITIAL CONDITIONS ARE SET ZERO
C
C     7.) FORCE CARDS (I5.1X.A6)
C
C       COL  1 TO  5     LAST NODE TO WHICH A FORCE IS TO BE SPECIFIED
C       COL  7 TO 12     MUST CONTAIN FORCE
C
C       SUBSEQUENT CARDS (I5.5X.6F10.0)
C
C       THE FOLLOWING VALUES ARE EACH INTERPRETTED AS FORCES IF THE
C       CORRESPONDING BOUNDARY CODE IS A 0 *ZERO* AND AS A DISPLACEMENT
C       IF THE CORRESPONDING BOUNDARY CODE IS 1 *ONE*.
C
C       COL  1 TO  5     NODE TO WHICH FORCE OR DISPLACEMENT IS APPLIED
C       COL 11 TO 20     VALUE OF 1 FORCE DISPLACEMENT
C       COL 21 TO 30     VALUE OF 2 FORCE DISPLACEMENT    * AS *
C       COL 31 TO 40     VALUE OF 3 FORCE DISPLACEMENT * REQUIRED *
C       COL 41 TO 50     VALUE OF 4 FORCE DISPLACEMENT
C       COL 51 TO 60     VALUE OF 5 FORCE DISPLACEMENT
C       COL 61 TO 70     VALUE OF 6 FORCE DISPLACEMENT
C
C
C      7.1) SURFACE LOAD CARDS (I5.1X.A6)
C
C       COL  1 TO  5     NUMBER OF LOADED FACE CARDS
C       COL  7 TO 12     MUST CONTAIN BLOADS
C
C       CARD 1. (I5.1X.A5.I4.8I5.8I3)
C
C       COL  1 TO  5     DIMENSION OF LOADING SURFACE. (1 OR 2).
C       COL  7 TO 11     SLD(NN). ALPHA-NUMERIC NAME OF SURFACE LOADING
C                        SUBROUTINE (NN IS BETWEEN 1 AND 5)
C       COL 12 TO 15     NET.NUMBER OF ADDITIONAL ELEMENT LOAD
C                        SURFACES TO BE GENERATED FROM CURRENT MODEL.
C       COL 16 TO 20     IFFE(NN). NODE NUMBERS DEFINING LOADING SURFACE
C       COL 21 TO 25     OF CURRENT ELEMENT.
C       COL ...          (IDENTIF (FROM 2 TO 8 AS REQUIRED
C       COL 51 TO 55
C       COL [illegible]  [illegible] INCREMENT VALUE ADDED TO IFFES(NN) TO
C       [illegible]
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C        COL ...          (IDENTIFY FROM 2 TO 8 AS REQUIRED)                        MAN532C
C                                                                                   MAN533C
C        CARD 2. (8F10.0)                                                           MAN534C
C                                                                                   MAN535C
C        COL  1 TO 80     LOAD AT NODES GIVEN ON PREVIOUS CARD *                    MAN536C
C                         MUST CORRESPOND IN SEQUENCE TO THE NODE NUMBERS           MAN537C
C                                                                                   MAN538C
C                                                                                   MAN539C
C                                                                                   MAN540C
C      7.2) ELEMENT LOAD CARDS (I5,1X,A6 )                                          MAN541C
C                                                                                   MAN542C
C        COL  1 TO  5     NLD. NUMBER OF ELEMENT LOAD CARDS.                        MAN543C
C        COL  7 TO 11     MUST CONTAIN ELOADS                                       MAN544C
C                                                                                   MAN545C
C        SUBSEQUENT CARDS (I5,1X,A5,I4,I5,6F10.0)                                   MAN546C
C                                                                                   MAN547C
C        COL  1 TO  5     IEL. INITIAL ELEMENT OF A GENERATED SEQUENCE.             MAN548C
C        COL  7 TO 11     ELM(NN). ALPHA-NUMERIC NAME OF ELEMENT                    MAN549C
C                         SUBROUTINE WHERE ELEMENT LOADS ARE COMPUTED.              MAN550C
C                         USED AS CHECK TO INSURE IEL, ETC. ARE PROPER              MAN551C
C                         ELEMENTS.                                                 MAN552C
C        COL 12 TO 15     INC. INCREMENT NUMBER IN A GENERATED SEQUENCE,            MAN553C
C                         (DEFAULT = 1).                                            MAN554C
C        COL 16 TO 20     JEL. TERMINAL ELEMENT NUMBER IN A GENERATED               MAN555C
C                         SEQUENCE. IF JEL = 0, ONLY IEL IS COUNTED.                MAN556C
C        COL 21 TO 80     PR-USER DEFINED VALUES FOR DETERMINING BODY               MAN557C
C                         LOADS IN THE ISW=5 PORTION OF ELM(NN).                    MAN558C
C                                                                                   MAN559C
C        NOTE. USER MUST PROVIDE COMPUTATION OF LOADS IN ELMTNN.                    MAN560C
C        PR IS TRANSFERED TO SUBROUTINE ELMTNN IN THE U VECTOR,                     MAN561C
C        WHEN ISW =5, ONLY.                                                         MAN562C
C                                                                                   MAN563C
C      7.3) PROPORTIONAL LOADS FOR TIME DEPENDENT ANALYSIS                          MAN564C
C                                                                                   MAN565C
C        TRANSFER TO THIS OPTION OCCUPS ONLY FOR TIME ANALYSES.                     MAN566C
C                                                                                   MAN567C
C        ONE CARD FOR EACH PROPORTIONAL LOAD REQUIRED                               MAN568C
C                                                                                   MAN569C
C        COL  1 TO  5     PROPORTIONAL LOAD TYPE, 1,2 OR 3                          MAN570C
C        COL  6 TO 10     F. TABLE CONSTANT                                         MAN571C
C        COL 11 TO 20     TMIN. SMALLEST TIME LOADING IS VALID                      MAN572C
C        COL 21 TO 30     TMAX. LARGEST TIME LOADING IS VALID                       MAN573C
C        COL 31 TO 40     A0                                                        MAN574C
C        COL 41 TO 50     A1                                                        MAN575C
C        COL 51 TO 60     A2                                                        MAN576C
C        COL 61 TO 70     A3                                                        MAN577C
C        COL 71 TO 80     A4                                                        MAN578C
C                                                                                   MAN579C
C        LOAD TYPE 1.   T = TIME                                                    MAN580C
C                                                                                   MAN581C
C            PROP = A0 + A1*T + A2*T*T + A3*T*T*T + A4*T*T*T*T                      MAN582C
C                                                                                   MAN583C
C        LOAD TYPE 2.                                                               MAN584C
C                                                                                   MAN585C
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C         PROP = A0*(SIN(A1*T))**K + A2*(COS(A3*T))**P + A5                    MAN586C
C                                                                              MAN587C
C      LOAD TYPE 3.                                                            MAN588C
C                                                                              MAN589C
C         PROP - USER DEFINED FUNCTION FROM SUBROUTINE                         MAN590C
C                EXPRLD(PROP.T.A)  WHERE A IS AN ARRAY WITH                    MAN591C
C                INFORMATION FOR COLUMNS 6-80 OF DATA CARDS.                   MAN592C
C                                                                              MAN593C
C      **NOTE** PROPORTIONAL LOADS CAN BE ACCUMULATED FROM DIFFERENT           MAN594C
C            TYPES AT THE SAME TIME.                                           MAN595C
C                                                                              MAN596C
C    8.) INITIATION OF TIME INDEPENDENT SOLUTION (I5.1X,A6)                    MAN597C
C                                                                              MAN598C
C      COL  1 TO  5     IOUT. OUTPUT CONTROL CODE.                             MAN599C
C                                                                              MAN600C
C                       IOUT .EQ. 0. ALL STRESSES AND DISP. PRINTED            MAN601C
C                       IOUT .NE. 0. SELECTED PRINTOUT. MORE DATA INPUT        MAN602C
C                       SEE SECTION 9 FOR DATA PREPARATION.                    MAN603C
C                                                                              MAN604C
C      COL  7 TO 12     MUST CONTAIN  SOLVE  *INDICATES ALL DATA INPUT*        MAN605C
C                       COMPLETE FORMULATION AND SOLUTION OF EQUATIONS.        MAN606C
C      COL  7 TO 12     MUST CONTAIN  RESOLV TO OBTAIN SUBSEQUENT              MAN607C
C                       SOLUTIONS WHERE BOUNDARY CODES DO NOT CHANGE           MAN608C
C                       AND ALL PRESCRIBED DISPLACEMENTS ARE ZERO.             MAN609C
C                                                                              MAN610C
C     8.1  INITIATION OF DYNAMIC SOLUTION BY EXPLICIT INTEGRATION.             MAN611C
C                                                                              MAN612C
C      COL  1 TO  5     IPRT.  OUTPUT CONTROL FOR DISPLACEMENT AND             MAN613C
C                       STRESS PRINTOUT. SEE SECT. 9 FOR DATA INPUT.           MAN614C
C                       IOUT = 1 - MIN(1.IPRT)                                 MAN614C
C                       IF IOUT .NE. 0. THE SPATIAL CONTROL DATA               MAN615C
C                       COMES AT THE END OF THE DYNAMIC SEGMENT.               MAN616C
C      COL  7 TO 12     MUST CONTAIN EXPLIC                                    MAN617C
C                                                                              MAN618C
C      SUBSEQUENT CARDS (2I5.2F10.0.2I5)                                       MAN619C
C                                                                              MAN620C
C      COL  1 TO  5     NUMBER OF TIME STEPS                                   MAN621C
C      COL  6 TO 10     PRINT INTERVAL                                         MAN622C
C      COL 11 TO 20     TIME INCREMENT                                         MAN623C
C      COL 21 TO 30     NEWMARK DELTA-DAMPING TERM (GAMMA - .5)                MAN624C
C      COL 31 TO 35     NUMBER OF TIME EVOLUTION ELEMENT VARIABLE PLOTS        MAN625C
C      COL 36 TO 40     NPROP. NUMBER OF PROPORTIONAL LOADS TO BE INPUT        MAN626C
C      COL 41 TO 45     NFORC. LAST NODE ON WHICH A FORCE IS CHANGED           MAN627C
C                       DURING EACH TIME STEP.                                 MAN628C
C      COL 46 TO 50     KKK. STABILITY CHECK OVERRIDE ** CAUTION USE           MAN629C
C                       ONLY WHEN A BETTER ESTIMATE OF THE STABLE TIME         MAN630C
C                       STEP IS AVAILABLE THAN CAN BE PERFORMED BY CODE        MAN631C
C                       KKK ZERO. USES INTERNAL STABILITY CHECK.               MAN632C
C                       KKK NONZERO. DISREGARDS STABILITY CHECK.               MAN633C
C      COL 51 TO 55     NONL.   0 FOR LINEAR                                   MAN633
C                               1 FOR NON LINEAR                               MAN633C
C      COL 56 TO 60     LPF.    0 PRINTS LOADS                                 MAN633C
C                               1 SUPPRESS PRINT                               MAN633C
                                                                               MAN634C
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C        SUBSEQUENT CARDS (3I5) ONE FOR EACH STRESS PLOT.                      MAN635C
C                                                                              MAN636C
C        COL  1 TO  5    ELEMENT NUMBER CONTAINING STRESS TO BE PLOTTED.       MAN637C
C        COL  6 TO 10    LOCAL COORDINATE POINT CODE, 1 TO 9, AS               MAN638C
C                        PATTERNED AFTER, COL 11 TO COL 19, IN SECT. 9.        MAN639C
C        COL 11 TO 15    PLOT COMPONENT CODE, 1 TO 6 FOR SIGMA(I,J),           MAN640C
C                        I.E., SIGMA(1,1)=1, SIGMA(1,2)=2, SIGMA(1,3)=3,       MAN641C
C                        SIGMA(2,2) = 4, SIGMA(2,3) = 5, SIGMA(3,3) = 6.       MAN642C
C                                                                              MAN643C
C        IF(NPROP.NE.0) READ PROPORTIONAL LOAD CARDS, SEE SECT. 7.3            MAN644C
C                                                                              MAN645C
C        IF(NFORC.NE.0) READ FORCE CARDS AT EACH TIME STEP. IF OUTPUT IS       MAN646C
C        LIMITED BY IOUT NONZERO, THE FIRST FORCE CARD SET PRECEDES            MAN647C
C        OUTPUT CARDS AND THE REMAINDER FOLLOW THE OUTPUT CARDS NO BLANK       MAN648C
C        CARDS MAY BE USED BETWEEN SETS OF CARDS OTHER THAN THE USUAL          MAN649C
C        BLANK TERMINATOR CARD FOR FORCE INPUT CARDS.                          MAN650C
C                                                                              MAN651C
C        IF(IOUT.NE.0)   DATA FOR SPATIAL PRINTOUT CONTROL, SEE SECT.9.        MAN652C
C                                                                              MAN653C
C                                                                              MAN654C
C       SPECIAL COMMENTS FOR DYNAMIC OPTION                                    MAN655C
C                                                                              MAN656C
C       (1) ONLY COLUMNS 1 TO 66 ARE AVAILABLE FOR PAGE HEADING.               MAN657C
C       (2) MAXIMUM ADVANTAGE OF ELEMENT REUSE OPTION SHOULD BE TAKEN.         MAN658C
C       (3) INITIAL CONDITIONS FOR DISPLACEMENT AND VELOCITY VECTORS,          MAN659C
C           AS WELL AS STORAGE FOR ACCELERATION VECTOR, MAY BE MADE            MAN660C
C           THROUGH INPUT OF AN INITIAL CONDITION CARD SET, WITHOUT            MAN661C
C           SPECIFIED INITIAL CONDITIONS THEY ARE AUTOMATICALLY SET ZERO.      MAN662C
C       (4) SPATIAL LOADING IS INPUT THROUGH FORCE OR BOUNDARY                 MAN663C
C           PRESSURE CARDS.                                                    MAN664C
C       (5) EXTREME CAUTION ON ORDER OF DATA CARDS MUST BE OBSERVED.  NO       MAN665C
C           EXTRA CARDS ARE PERMITTED AND STRICT COUNTS ARE OBSERVED           MAN666C
C           EXCEPT FOR THE NUMBER OF FORCE CARDS USED IN EACH TIME STEP.       MAN667C
C                                                                              MAN668C
C       8.2) INITIATION OF IMPLICIT TIME INTEGRATIONS (I5,1X,A6)               MAN669C
C                                                                              MAN670C
C        COL  1 TO  5    NSEQ, NUMBER OF TIME SEQUENCES                        MAN671C
C        COL  7 TO 12    MUST CONTAIN VISCOE FOR LINEAR VISCOELASTIC           MAN672C
C                        QUASI-STATIC PROBLEMS (ONE INITIAL CONDITION          MAN673C
C                        ONLY MUST BE USED)                                    MAN674C
C        COL  7 TO 12    MUST CONTAIN IMPLIC FOR DYNAMIC IMPLICIT              MAN675C
C                        INTEGRATIONS (THREE INITIAL CONDITIONS ARE            MAN676C
C                        REQUIRED, MORE CAN BE SPECIFIED WITHOUT ERROR)        MAN677C
C                                                                              MAN688C
C        SUBSEQUENT CARDS, ONE SET FOR EACH TIME SEQUENCE                      MAN689C
C                                                                              MAN690C
C        CARD 1, (F10.0,8I5,2F10.0,2I5)                                        MAN691C
C                                                                              MAN692C
C        COL  1 TO 10    DT, TIME INCREMENT (NONZERO FOR IMPLIC)               MAN693C
C        COL 11 TO 15    NTS, NUMBER OF TIME STEPS IN SEQUENCE                 MAN694C
C        COL 16 TO 20    INT, PRINT INTERVAL (DEFAULT 1)                       MAN695C
C        COL 21 TO 25    NNI, FIRST NODE PRINTED                               MAN696C
C        COL 26 TO 30    NNE, LAST  NODE PRINTED                               MAN697C
C        COL 31 TO 35    NEI, FIRST ELEMENT STRESS TO BE PRINTED               MAN698C
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C        COL 36 TO 40    NEF. LAST  ELEMENT STRESS TO BE PRINTED                    MAN699C
C        COL 41 TO 45    NPROP. NUMBER OF PROPORTIONAL LOADS IN SEQUENCE            MAN700C
C        COL 46 TO 50    NFORC. LAST NODE FOR GENERALIZED FORCES TO BE              MAN701C
C                        INPUT FOR EACH TIME IN SEQUENCE (SEE SECT.7.               MAN702C
C                        FOR DATA PREPARATION FORMATS)                              MAN703C
C        COL 51 TO 60    BETA. NEWMARK INTEGRATION PARAMETER (IMPLIC)               MAN704C
C                        DEFAULT VALUE IS 0.25                                      MAN705C
C        COL 61 TO 70    DEL = GAMMA - 0.5. NEWMARK INTEGRATION                     MAN706C
C                        PARAMETER (IMPLIC)                                         MAN707C
C        COL 71 TO 75    NUMPLT. NUMBER OF STRESS PLOTS REQUESTED                   MAN708C
C                        (INPUT ON FIRST TIME SEQUENCE ONLY)                        MAN709C
C        COL 76 TO 80    NITS. NUMBER OF ITERATIONS PER TIME STEP                   MAN710C
C                        (TO ACHIEVE EQUILIBRIUM IN NON LINEAR PROBLEMS)            MAN711C
C                        ZERO OR BLANK FOR LINEAR PROBLEMS.                         MAN712C
C                        POSITIVE - REFORM THE NEWTON JACOBIAN MATRIX               MAN713C
C                                   AT START OF EACH TIME STEP ONLY.                MAN714C
C                        NEGATIVE - REFORM THE NEWTON JACOBIAN MATRIX               MAN715C
C                                   AT EACH ITERATION IN EACH TIME STEP.            MAN716C
C                        IF CONVERGENCE OCCURS BEFORE ABS(NITS)                     MAN717C
C                        ITERATIONS. THE PROGRAM WILL PROCEED TO THE                MAN718C
C                        NEXT TIME STEP.                                            MAN719C
C                        NOTE * * BECAUSE OF THE CONVERGENCE TEST USED.             MAN720C
C                                 A MINIMUM OF TWO ITERATIONS MUST BE               MAN721C
C                                 PERFORMED BEFORE THE CHECK CAN BE                 MAN722C
C                                 IMPOSED.  THUS. USE OF NITS CAN BE                MAN723C
C                                 INEFFICIENT IF LINEAR PROBLEMS ARE                MAN724C
C                                 BEING SOLVED.                                     MAN725C
C                                                                                   MAN726C
C        CARD 2. (2I5.7F10.0)                                                       MAN727C
C                                                                                   MAN728C
C        ONE FOR EACH NPROP. SEE SECT.7.3 FOR DATA PREPARATION                      MAN729C
C                                                                                   MAN730C
C                                                                                   MAN731C
C        SUBSEQUENT CARDS FOR EACH TIME STEP IN THE SEQUENCE                        MAN732C
C                                                                                   MAN733C
C        FORCE CARDS.  SEE SECTION 7. FOR DATA PREPARATION FORMATS.                 MAN734C
C                                                                                   MAN735C
C       8.3) MESH CHECKS AND PLOTS                                                  MAN736C
C                                                                                   MAN736C
C        COL  7 TO 10    MUST CONTAIN MESH.                                         MAN736C
C                        PROGRAM WILL CHECK THE PREVIOUSLY INPUTTED MESH            MAN736C
C                        FOR ERRORS. BUT DOES NOT PROCEED TO SOLVE.                 MAN736C
C        COL  7 TO 10    MUST CONTAIN PLOT.                                         MAN736C
C                        PERFORMS SAME CHECKS AS FOR MESH AND PLOTS THE             MAN736C
C                        MESH. (GDS OR NOVA PLOT ROUTINES AVAILABLE).               MAN736C
C                                                                                   MAN736C
C      8.4) INPUT OF FOURIER SERIES HARMONICS                                       MAN736C
C                                                                                   MAN736C
C        COL  1 TO  5    IOUT. (SEE 8.)                                             MAN736C
C        COL  7 TO 12    FOURIE                                                     MAN736C
C                                                                                   MAN736C
C        CARD 1.  (I5.5X.6F10.0)                                                    MAN736C
C                                                                                   MAN736C
C        COL  1 TO 10    HARMONIC NUMBER                                            MAN736C
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C        COL 11 TO 20     1 - FORCE/DISPL. MULTIPLIER                               MAN736C
C        COL 21 TO 30     2 - FORCE/DISPL. MULTIPLIER                               MAN736C
C        COL 31 TO 40     3 - FORCE DISPL. MULTIPLIER                               MAN736C
C        COL 41 TO 50     4 - FORCE USER MULTIPLIER                                 MAN736C
C        COL 51 TO 60     5 - FORCE USER MULTIPLIER                                 MAN736C
C        COL 61 TO 70     6 - FORCE USER MULTIPLIER                                 MAN736C
C        COL 71 TO 80     USER CONSTANT.                                            MAN736C
C                                                                                   MAN736C
C                                                                                   MAN736C
C     9.) OUTPUT CONTROL FOR LIMITED PRINTS                                         MAN737C
C                                                                                   MAN738C
C        DISPLACEMENT OUTPUT CONTROL. IF IOUT .NE. 0.                               MAN739C
C                                                                                   MAN740C
C        CARD 1.  (I5)                                                              MAN741C
C                                                                                   MAN742C
C        COL  1 TO  5     NUMDIS - NUMBER OF DISPLACEMENT PRINT CARDS               MAN743C
C                                                                                   MAN744C
C        SUBSEQUENT CARDS (2I5) SKIP IF NUMDIS = 0                                  MAN745C
C                                                                                   MAN746C
C        COL  1 TO  5     NODAL NUMBER TO BE OUTPUT.                                MAN747C
C        COL  6 TO 10     HIGHER NODE NUMBER OF A GENERATED SEQUENCE,               MAN748C
C                         IF ZERO JUST FIRST NODE IS COUNTED.                       MAN749C
C        COL 11 TO 15     INCREMENT TO GENERATOR. DEFAULT = 1                       MAN750C
C                         *** REPEAT UNTIL.NUMDIS CARDS HAVE BEEN READ              MAN751C
C                                                                                   MAN752C
C        STRESS OUTPUT CONTROL. IF IOUT .NE. 0.                                     MAN453C
C                                                                                   MAN754C
C        CARD 1.  (I5.5X,9I1)                                                       MAN755C
C                                                                                   MAN756C
C        COL  1 TO  5     NUMSTR - NUMBER OF STRESS OUTPUT CARDS                    MAN757C
C        COL 11 TO 19     NSIG(9) - PRINT PATTERN WITHIN AN ELEMENT.                MAN758C
C                         LOCAL POINTS OF EACH ELEMENT CAN BE                       MAN759C
C                         SUPRESSED BY NON-ZERO ENTRIES AS FOLLOWS.                 MAN760C
C                                                                 E.G.              MAN761C
C             COL   11    SUPPRESS PRINT AT LOCAL POINT 1. ( 0. 0. 0)               MAN762C
C             COL   12    SUPPRESS PRINT AT LOCAL POINT 2. (-1. 0. 0)               MAN763C
C             COL   13    SUPPRESS PRINT AT LOCAL POINT 3. ( 1. 0. 0)               MAN764C
C             COL   14    SUPPRESS PRINT AT LOCAL POINT 4. ( 0.-1. 0)               MAN765C
C             COL   15    SUPPRESS PRINT AT LOCAL POINT 5. ( 0. 1. 0)               MAN766C
C             COL   16    SUPPRESS PRINT AT LOCAL POINT 6. ( 0. 0.-1)               MAN767C
C             COL   17    SUPPRESS PRINT AT LOCAL POINT 7. ( 0. 0. 1)               MAN768C
C             COL   18    SUPPRESS PRINT AT LOCAL POINT 8                           MAN769C
C             COL   19    SUPRESS PRINT AT LOCAL POINT 9                            MAN770C
C                                                                                   MAN771C
C        SUBSEQUENT CARDS (2I5) SKIP IF NUMSTR = 0                                  MAN772C
C                                                                                   MAN773C
C        COL  1 TO  5     ELEMENT NUMBER TO BE PRINTED.                             MAN774C
C        COL  6 TO 10     HIGHER ELEMENT NUMBER OF A GENERATED SEQUENCE,            MAN775C
C                         IF ZERO ONLY FIRST ELEMENT IS COUNTED.                    MAN776C
C        COL 11 TO 15     INCREMENT TO GENERATOR. DEFAULT = 1                       MAN777C
C                         *** REPEAT UNTIL.NUMSTR CARDS HAVE BEEN READ              MAN778C
C                                                                                   MAN779C
C                                                                                   MAN780C
C***********************************************************************************MAN781C
```

C. Listing of the Contact/Impact Subroutines Added to FEAP 74

The listings for the subroutines which are added to FEAP 74 for the contact/impact theory described herein are given below.

```
              SUBROUTINE CONTAC(ISW,IX,NEL1,NDF,NUMNP,DU,U,UD,UDD)
        C******************** CONTAC ***** 04/29/74 *****************************************
000014        LOGICAL CFLAG,FLAGC
000014        DIMENSION IX(NEL1,NUMNP),DU(NDF,NUMNP),U(NDF,NUMNP),UD(NDF,NUMNP),
             C          UDD(NDF,NUMNP)
000014        DIMENSION RMP(10)
000014        COMMON/CNTACT/FLAGC,CFLAG,LIST,ICLIST(10),ICDEG(10),PU1,PU2,
             C     RM(2,10)
000014        COMMON/LABELS/HEAD(12),O,IPG,XHED(3),UHED(6),XH,FH,UH,NSTR,FLAG(7)
000014        COMMON/TAPES/ ITP5,ITP6
000014        DATA CFLAG,FLAGC /.FALSE.,.FALSE./
000014        GO TO (1,2,3),ISW
000022  1     READ(ITP5,1000) LIST,PU1,PU2
000034        PP = PU1+PU2
000036        IF(PP.EQ.0.0) PP = 1.
000044        READ(ITP5,1001) (ICLIST(L),ICDEG(L),L=1,LIST)
000070        WRITE(ITP6,2000) O,HEAD,IPG,PU1,PU2,(ICLIST(L),ICDEG(L),L=1,LIST)
000131        IPG = IPG + 1
000133        FLAGC = .TRUE.
000134        RETURN
000134  2      DO 200 L = 1,LIST
000141        N = ICLIST(L)
000143        IDEG = ICDEG(L)
000144        I = IX(1,N)
000150        RM(1,L) = DU(IDEG,I)
000155        I = IX(3,N)
000161  200   RM(2,L) = DU(IDEG,I)
000171        WRITE(ITP6,2002)
000175        DO 220 L = 1,LIST
000202        N = ICLIST(L)
000204        RMP(L) = RM(1,L) + RM(2,L)
000210        IF(RM(1,L).EQ.0.0.OR.RM(2,L).EQ.0.0) RMP(L) = RMP(L)*1.E+30
000222  220   WRITE(ITP6,2001) N,(RM(I,L),I=1,2),RMP(L)
000256        DO 210 N = 1,NUMNP
000260        DO 210 I = 1,NDF
000261  210   DU(I,N) = 0.0
000272        RETURN
000272  3     DO 300 L = 1,LIST
000274        N=ICLIST(L)
000276        IDEG = ICDEG(L)
000277        K = IX(2,N)
000303        UD(IDEG,K) = 0.0
000310        UDD(IDEG,K) = 0.0
000313        IF(U(IDEG,K)) 300,300,310
000317  310   I = IX(1,N)
000323        J = IX(3,N)
000326        CU =(PU1*U(IDEG,I)+PU2*U(IDEG,J))/PP
000342        [illegible]
000347        [illegible]
000352        [illegible] RM(1,L)*[illegible](IDEG,I)+RM(2,L)*[illegible](IDEG,J))/RMP(L)
[illegible]   IF [illegible] UD(IDEG,K) [illegible] RM(2,L)*UD(IDEG,J)-CU)
[illegible]   IF(RM [illegible] UDD(IDEG,K) [illegible] +RM(1,L)*UDD(IDEG,J)-CU)
[illegible]
```

```
   451          UDD(IDEG.J) = CU
 .0454   300    CONTINUE

000457          RETURN

000457   1000   FORMAT(I5.5X.2F10.0)
000457   1001   FORMAT(2I5)
000457   2000   FORMAT(A1.12A6.30X.4HPAGE.I4//5X.25HCONTACT SHEET DESCRIPTION//
               C      10X.13HBODY 1 RO*U =.E13.5/
               C      10X.13HBODY 2 RO*U =.E13.5//
               X   11X.7HELEMENT.1X.9HDIRECTION /(I15.I10))
000457   2001   FORMAT(I10.6E13.4)
000457   2002   FORMAT( 10H   ELEMENT.13H  BODY 1 MASS.13H  BODY 2 MASS)
000457          END
```

```
                SUBROUTINE ELMT05(N,MA,NDIM,NDF,NEL,NEL1,NSTF,NSIZV,NVEC,MCT,DM,D,        ELM  1C
               X XYZ,IX,F,FORCE,ESTIF,U,VECT,ISW)
          C****************** ELMT05 ***** 04/29 74 ****************************************
000027          LOGICAL NPR
000027          DIMENSION ESTIF(NSTF,NSTF),FORCE(NSTF,2)
000027          DIMENSION U(NDF,1),IX(NEL1,1),PLOT(8),THED(8)
000027          COMMON/LOCALS/ DUL(6,20),UL(6,20),UDL(6,20),UDDL(6,20)
000027          COMMON/PRTPLT/ NSIG(9),NPLT(9,2),NT,NSTEP,NUMPLT,NEDATA(20,3),NPR
000027          COMMON/SHAP/ XJAC,SHAPE(4,20),SG(3,3),SH(3,3),X(3,20)
000027          COMMON/TAPES/ ITP5,ITP6
000027          COMMON/TIMHIS/ TIME,DT,DTP,NH,ISZH,C0,C1,C2,C3,C4,C5,C6,MCT
000027          EQUIVALENCE (PLOT,TAU),(PLOT(2),ETA)
000027          DATA THED/3HTAU,3HETA,2HU1,3HUD1,4HUDD1,2HU3,3HUD3,4HUDD3/
000027          GO TO (1,2,3,3,5,3), ISW
000041    1     READ(ITP5,1000) IDEG,NPS
000051          IDEG = MAX0(IDEG,1)
000055          WRITE(ITP6,2000) IDEG,NPS
000064          TOL = 1.E-06
000066          I = NDF  + IDEG
000073          J = I + NDF
000074          RI = 0.5
000075          RJ = 0.5
000076          C6 = 0.
000077          NV = NPS
000100          NA = NPS + NPS
000101          RETURN
000102    2     DM = X(IDEG,3) - X(IDEG,1)
000106          DM = DM + 1.E-25
000110          RETURN
000110    3     TAU = UL(IDEG,2)
000112          DD = (X(IDEG,3)-X(IDEG,1)) + (UL(IDEG,3)-UL(IDEG,1))
000120          ETA = 0.
000121          IF(DD.LT.TOL) ETA = 1.
000125          IF(TAU.LT.0.0  ) ETA = 0.0
000130          IF(ISW.GT.3) GO TO 4
000134          ESTIF(IDEG,I) = ETA
000142          ESTIF(I,IDEG) = ETA
000145          ESTIF(I,J) = -ETA
000151          ESTIF(J,I) = -ETA
000155          ESTIF(I,I) = 1.0 - ETA
000163          RETURN
000163    4      IF(ISW.EQ.4) GO TO 6
000166          FORCE(I,1) = ETA*DD
000174          IF(ETA.EQ.0.0) FORCE(I,1) = - TAU
000203          TAU = AMAX1(0.0,TAU)
000207          FORCE(IDEG,1) = -TAU
000215          FORCE(J,1) = TAU
000220    6     IF(.NOT.NPR) WRITE(ITP6,2001) N,DM,MA,TAU,ETA
000243          IF(NUMPLT.LE.0) RETURN
000246          PLOT(3) = UL(IDEG,1)
000250          PLOT(4) = UDL(IDEG,1)
[illegible]     PLOT(5) = UDDL(IDEG,1)
[illegible]     PLOT(6) = UL(IDEG,3)
```

```
 .256          PLOT(7) = UDL(IDEG.3)
 J0260         PLOT(8) = UDDL(IDEG.3)
 000262        DO 60 K = 1.NUMPLT
 000263        KK = NPLT(K.2)
 000265        IF(NPLT(K.1).GT.0) CALL FLDATA(NDIM.NPLT(K.1).THED(KK).X(1.2).
             C    PLOT(KK))
 000306   60   CONTINUE
 000311   5    RETURN
 000312   1000 FORMAT(2I5)
 000312   2000 FORMAT(5X.21HPOINT CONTACT ELEMENT.
             C         5X.27HCONTACT DEGREE OF FREEDOM = I3.5X.5HNPS = I5)
 000312   2001 FORMAT(5X.7HELEMENT.I5.5X.A5.5X.8HMATERIAL.I3.5X.5HTAU =.E15.5.5X.
             X    5HETA =.F3.1)
 000312        END
```

```
                SUBROUTINE ELMT09(N.MA.NDIM.NDF.NELM.NEL1.NSTF.NSIZV.NVEC.MCT.DM.      ELM  1C
               X D.XYZ.IX.F.FORCE.ESTIF.U.VECT.ISW)                                    ELM  2C
000027          LOGICAL NOPRNT.NPR.NPL
000027          DIMENSION ESTIF(NSTF.NSTF).D(3.21.1).IX(NEL1.1).U(NDF.1).              ELM  4C
               X V(3.20).DU(3).XX(3).FORCE(NSTF.2)                                     ELM  5C
000027          COMMON/GAUS/ LIM.SGAUSS(5.5).WGAUSS(5.5)                               ELM  3C
000027          COMMON/LABELS/HEAD(12).O.IPG.XHED(3).UHED(6).XH.FH.UH.NSTP.FLAG(7)
000027          COMMON/LOCALS/ DUL(6.20).UL(6.20).UDL(6.20).UDDL(6.20)
000027          COMMON/PRTPLT/ NSIG(9).NPLT(9.2).NT.NSTEP.NUMPLT.NEDATA(20.3).NPP
000027          COMMON/SHAP/ XJAC.SHAPE(4.20).SG(3.3).SK(3.3).X(3.20)
000027          COMMON/TAPES/ ITP5.ITP6                                                ELM 10C
000027          COMMON/TIMHIS/ TIME.DT.DTP.NH.ISZH.C0.C1.C2.C3.C4.C5.C6.NCT
000027          DATA SH.EH.BL/6HSTRESS.6HSTRAIN.6H                                     ELM 11C
000027          GO TO (1.2.3.4.5.4) . ISW
000041    1     CONTINUE                                                               ELM 13C
          C.... BAR STIFFNESS CHARACTERIZATION                                         ELM 14C
000041          READ(ITP5.1000) E.AA.RO
000053          WRITE(ITP6.2100) E.AA.RO
000065          D(1.1.MA) = E                                                          ELM 17C
000075          D(1.2.MA)=AA
000100          D(1.3.MA)=RO
000103          C6 = 0.
000104          RETURN                                                                 ELM 20C
000104    2     CONTINUE                                                               ELM 21C
000104          RETURN                                                                 ELM 22C
          C.... COMPUTATION OF BAR STIFFNESS NO BENDING * ISOPARAMETRIC 1ST AND 2N     ELM 23C
000105    3     CONTINUE                                                               ELM 24C
          C.... CONSTRUCT STIFFNESS AT INTEGRATION POINTS                              ELM 25C
000105          EA = D(1.1.MA)*D(1.2.MA)                                               ELM 26C
000113          RA = D(1.3.MA)*D(1.2.MA)                                               ELM 27C
000116          DO 260 II = 1.NELM                                                     ELM 28C
000120          SS = SGAUSS(II.NELM)                                                   ELM 29C
000123          CALL LINE(SS.NDIM.NELM)                                                ELM 30C
000125          WW=WGAUSS(II.NELM)
000134          DVOL=EA*WW/(XJAC*XJAC*XJAC)
          C.... COMPUTE A LUMPED MASS MATRIX                                           ELM 32C
000137          IU = 0                                                                 ELM 33C
000140          RAA=RA*XJAC*WW
000142          DO 217 I = 1.NELM                                                      ELM 34C
000143          AA=RAA*SHAPE(2.I)
000146          DO 215 KK = 1.NDIM                                                     ELM 35C
000150   215    FORCE(IU+KK.2) = FORCE(IU+KK.2) + AA                                   ELM 37C
000152   217    IU = IU + NDF                                                          ELM 38C
000166          DO 250 KK = 1.NDIM                                                     ELM 39C
000167          DO 250 LL = KK.NDIM                                                    ELM 40C
000170          SGD=SG(KK.LL)*DVOL
000174          I1 = KK                                                                ELM 41C
000175          DO 240 I = 1.NELM                                                      ELM 42C
000177          SHI=SHAPE(1.I)*SGD
000200          L = 1                                                                  ELM 43C
000203          IF(KK.EQ.LL) L = I                                                     ELM 44C
000204          J1 = LL + NDF*(L-1)                                                    ELM 45C
000205          DO 230 J = L.NELM                                                      ELM 46C
```

```
  .14          ESTIF(I1.J1)=ESTIF(I1.J1)+SHI*SHAPE(1.J)
 .J225   230   J1 = J1 + NDF                                                  ELM 48C
 .00231  240   I1 = I1 + NDF                                                  ELM 49C
 000234  250   CONTINUE                                                       ELM 50
 000241  260   CONTINUE                                                       ELM 51
       C.....  CONSTRUCT SYMMETRIC PART OF STIFFNESS MATRIX                   ELM 5
 000243        KK = NELM*NDF                                                  ELM 53C
 000245        LL = KK - 1                                                    ELM 54C
 000247        DO 360 I=1.LL                                                  ELM 55C
 000250        K = I + 1                                                      ELM 56
 000252        DO 360 J = K.KK                                                ELM 57C
 000254        ESTIF(I.J) = ESTIF(I.J) + ESTIF(J.I)                           ELM 58C
 000266  360   ESTIF(J.I) = ESTIF(I.J)                                        ELM 59C
 000302        IF(C6.EQ.0.0) RETURN
 000304        DO 370 I = 1.NSTF
 000306  370   ESTIF(I.I) = ESTIF(I.I) + C6*FORCE(I.2)
 000324        RETURN                                                         ELM 60C
 000325  4     CONTINUE                                                       ELM 61C
 000325        E = D(1.1.MA)                                                  ELM 64C
 000331        NELP = NELM
 000332        DO 400 NN = 1.NELP
 000334        NOPRNT=NPR
 000336        IF(NSIG(NN).GT.0) NOPRNT= TRUE.
 000341        NPL=.TRUE.
 000342        IF(NPLT(NN.1).GT.0) NPL = .FALSE.
 000345        SS = SGAUSS(NN.NELP)
 000351        CALL LINE(SS.NDIM.NELM)                                        ELM 71C
       C....   COMPUTE STRESS AND STRAIN                                      ELM 72C
 000353        DO 100 KK = 1.NDIM                                             ELM 73C
 000360        XX(KK) = 0.0                                                   ELM 74C
 000361        DU(KK)= 0.                                                     ELM 75C
 000362        DO 100 LL = 1.NELM                                             ELM 76C
 000364        XX(KK) = XX(KK) + X(KK.LL)*SHAPE(2.LL)                         ELM 77C
 000372  100   DU(KK) = DU(KK) +UL(KK.LL)*SHAPE(1.LL)
 000404        EPS = 0.                                                       ELM 79C
 000405        DO 200 KK = 1.NDIM                                             ELM 80C
 000406  200   EPS = EPS + SK(KK.1)*DU(KK)/XJAC/XJAC                          ELM 81C
 000416        SIG = E*EPS                                                    ELM 82C
 000420        IF(NOPRNT) GO TO 350
 000421        MCT = MCT - 1                                                  ELM 83C
 000423        IF(MCT.GT.0) GO TO 300                                         ELM 84C
 000426        MCT=50
 000426        WRITE(ITP6.2000) HEAD.TIME.IPG.( XHED(I).XH.I=1.NDIM).BL.SH.BL.EH
 000465        IPG = IPG + 1                                                  ELM 87C
 000467    300 WRITE(ITP6.2001) N.MA.DM.(XX(I).I=1.NDIM).SIG.EPS              ELM 88C
 000523  350   IF(.NOT.NPL) CALL PLDATA(NDIM.NPLT(NN.1).SHAXIAL.XX.SIG)
 000542        IF(ISW.EQ.4) GO TO 400
 000545        WT = WGAUSS(NN.NELM)*D(1.2.MA)
 000553        WW = WT*D(1.3.MA)*XJAC
 000557        IU = 0
 000560        DO 400 I = 1.NELM
 000561        H  = SHAPE
 000565        EP = SHAPE
```

```
000573   615   FORCE(KK+IU.I) = FORCE(KK+IU.I) - AA*UDPL(KK.I) - BB*S(KK.I)
000615   600   IU = IU + NDF
000621   400   CONTINUE                                                  ELM 31
000624   5     RETURN
000625   1000  FORMAT(3F10.0)                                            ELM 3
000625   2000  FORMAT(1H1.12A6.E13.5.I2 .4HPAGE.I3 /10H   ELEMENT. 2HMATE
              X 3X.5(2A5))
000625   2001  FORMAT(I10.I5.5X.A5.5E12.4)                               ELM 360
000625   2100  FORMAT(37HOLINEAR ELASTIC MATERIAL. BAR ELEMENT//
              X        5X.3HE =.E15.5.5X.6HAREA =.E15.5.5X.9HDENSITY =E15.5)   ELM 370
000625   4000  FORMAT(A5.I5.1P3E12.5.24X.I10)                            ELM 380
000625         END                                                       ELM 390
```

```
              SUBROUTINE LINE(S,NDIM,NEL)
        C***** LINE        ******  1/02/73 **************************************
        C.... SHAPE FUNCTION ROUTINE FOR 1ST AND 2ND ORDER ISOPARAMETRIC LINES
000006        COMMON SHAP  XJAC,SHAPE(1,20),SG(3,3),SK(3,3),X(3,20)
        C.... FORM SHAPE FUNCTIONS
000006        SHAPE(1,1) = -0.5
000007        SHAPE(2,1) = 0.5*(1.0-S)
000012        SHAPE(1,NEL) = 0.5
000014        SHAPE(2,NEL) = 0.5*(1.0+S)
000016        IF(NEL.EQ.2) GO TO 350
000020        SHAPE(1,2) = -2.0*S
000021        SHAPE(2,2) = 1.0-S*S
000024        K = NEL-1
000026        DO 100 I = 1,NEL,K
000027        SHAPE(1,I) = SHAPE(1,I) - 0.5*SHAPE(1,2)
000033  100   SHAPE(2,I) = SHAPE(2,I) - 0.5*SHAPE(2,2)
        C.... FORM JACOBIAN MATRIX AND DETERMINANT
000040  350   DO 360 I = 1,3
000042  360   SK(I,1) = 0.0
000046        DO 400 I = 1,NDIM
000047        DO 400 J = 1,NEL
000050  400   SK(I,1) = SK(I,1) + X(I,J)*SHAPE(1,J)
000063        XJAC = 0.0
000064        DO 500 I = 1,NDIM
000065        XJAC = XJAC + SK(I,1)*SK(I,1)
000070        DO 500 J = 1,NDIM
000071  500   SG(I,J) = SK(I,1)*SK(J,1)
000103        XJAC = SQRT(XJAC)
000105        RETURN
000105        END
```

```
              SUBROUTINE NLNORM(ISW,MDEG,NN,U,DU,DF,F,PROP,NCT,NTT,IDEST,IBLK)
        C***** NLNORM    ****** 12/14/73   ****************************************
000017        DIMENSION U(1),DU(MDEG),DF(1),F(MDEG),IDEST(1)
000017        COMMON/NORMS/ UNORM,DNORM,ANORM,CS,CSP,DP,DNP,IFL,XFLAG,ERR
000017        GO TO (100,200,200,100), ISW
000026    100 CS=1.0
000027        CSP=1.0
000030        DP=0.75
000032        IFL=0
000032        XFLAG=.FALSE.
000033        ERR=1.0E-3
000035        RETURN
000036    200 UNORM = 0.
000037        DNORM = 0.
000037        ANORM=0.
000041        DO 500 N = 1,MDEG
000042        DUN = DU(N)
000044        UN = U(N)
000046        IF(ISW.EQ.3) UN = UN + DF(N) + DUN
000053        ANORM=ANORM+UN*DUN
000056        UNORM=UNORM+UN*UN
000060    500 DNORM=DNORM+DUN*DUN
000065        UNORM=SQRT(UNORM)
000067        DNORM=SQRT(DNORM)
000070        IF(NN.EQ.1) DNP=UNORM
000077        CS=1.0
000100        IF(UNORM*DNORM.NE.0.) CS=ANORM/(UNORM*DNORM)
000105        IF(IFL.EQ.1) DP=0.75
000111        IF(IFL.EQ.1.AND.XFLAG) CSP=CS
000120        IFL=0
000121        IF(DNORM.LE.0.5*UNORM) GO TO 550
000124        IFL=1
000125        DP=0.5*UNORM/DNORM
000127    550 IF(DP*DNORM.GT.DNP) DP=DNP/DNORM
000134        IF(DP.EQ.0.0) DP = 1.0
000136        IF(CS*CSP.LT.0.) DP=DP/2.
000142        IF(CS*CSP.GT.0.) DP=1.25*DP
000146        DNP=DP*DNORM
000150        CSP=CS
000150        IF(ISW.EQ.2) RETURN
000153        DO 600 N = 1,MDEG
000155        DF(N) = DF(N) + DU(N)
000160        IF(NCT.EQ.0) DF(N) = DU(N)
000164    600 DU(N) = DF(N)
000171        RETURN
000171        END
```

```
              SUBROUTINE TSOLVE(NUMNP.NUMEL.NUMMAT.NDIM.NDF.NEN.NEL1.NSTF.NVEC.    TSO  10
             1   NSIZV.NICD.NSICD.IBLK.ISZA.NEQB.MAXBAN.NDES.ID1.MB.IBUF.DS.       TSO  20
             2   TYPE.D.ICOD.XYZ.F.IX.IDEST.VECT.FORCE.ESTIF.LD.A.DU.MAXB.H.U.DF.  TSO  30
             3   NSEQ.TYME)                                                        TSO  40
       C***** TSOLVE     ****** 03 06/74  *****************************************  TSO  50
000053        LOGICAL CFLAG.FLAGC
000053        LOGICAL NPR VFLAG.DFLAG                                              TSO  60
000053        REAL LABL(12)
000053        DIMENSION TYPE(1).ICOD(1).XYZ(NDIM.1).F(NDF.1).IX(NEL1.1).           TSO  80
             1   IDEST(NDF.1).VECT(NSIZV.1).FORCE(NSTF.2).ESTIF(NSTF.NSTF).        TSO  90
             2   A(NEQB.1).DU(NDF.1).H(IBUF).U(NDF.1).DF(1).DS(MB).TYME(7).LD(1)   TSO 100
000053        COMMON/CNTACT FLAGC.CFLAG.LIST.ICLIST(10).ICDEG(10).RU1.RU2.
             C      RM(2.10)
000053        COMMON/LABELS/HEAD(12).O.IPG.XHED(3).UHED(6).XH.FH.UH.NSTR.FLAG(7)   TSO 110
000053        COMMON/LOCALS/ DUL(6.20).UL(6.20).UDL(6.20).UDDL(6.20)               TSO 120
000053        COMMON/NORMS/ UNORM.DNORM.ANORM.CS.CSP.DP.DNP IFL.XFLAG.EPR          TSO 130
000053        COMMON/PRTPLT/ NSIG(9).NPLT(9.2).NT.NSTEP.NUMPLT.NEDATA(20.3).NPR    TSO 140
000053        COMMON/SHAP/ XJAC.SHAPE(4.20).SG(3.3).SK(3.3).X(3.20)                TSO 150
000053        COMMON/TAPES/ ITP5.ITP6                                              TSO 160
000053        COMMON/TIMHIS/ TIME.DT.DTP.NH.ISZH.C0.C1.C2.C3.C4.C5.C6.NCT          TSO 170
000053        COMMON/VTAPE/ ITP13.ITP14.ITPD.ITUP                                  TSO 180
000053        DATA AWORD1.AWORD2/8H   F13.4..8H   E13.4./.AWORD3/8H   F12.2./
000053        DATA VH.AH.AWORD4.AWORD5/6H VEL. .6H ACCEL .6H(/12X..8H9E13.4))/
000053        DATA LABL(1)/8H (I12/                                                TSO 200
000053        DATA CFLAG.FLAGC/.FALSE...FALSE./
       C....  SUBROUTINE TO PERFORM IMPLICIT INTEGRATION WITH 1 OR 3 INIT. COND.   TSO 210
000053        DO 2 N=1.NEN                                                         TSO 220
000054        DO 2 J=1.NDF                                                         TSO 230
000055        DUL(J.N)=0.                                                          TSO 240
000060        UL(J.N) = 0.0                                                        TSO 250
000063        UDL(J.N)=0.                                                          TSO 260
000065 2      UDDL(J.N) = 0.0                                                      TSO 270
000073        DO 3 I=2.7                                                           TSO 280
000074 3      NSIG(I)=1                                                            TSO 290
000077        IF(IBLK.EQ.0) GO TO 4                                                TSO 300
000101        KU = NUMNP NICD                                                      TSO 310
000104        REWIND 7                                                             TSO 320
000106        WRITE(7) DS                                                          TSO 330
       C....  SET THE OUTPUT LABELS FOR DISPLACEMENT PRINTS                        TSO 340
000117 4      HEAD(12) = 6H TIME=                                                  TSO 350
000124        IFLG =-NSICD                                                         TSO 360
000126        MDEG = NSICD                                                         TSO 370
000127        MDEG2 = MDEG + MDEG                                                  TSO 380
000130        I = 2                                                                TSO 390
000131        DO 5 N = 1.NDIM                                                      TSO 400
000132        LABL(I) = AWORD1                                                     TSO 410
000134 5      I = I + 1                                                            TSO 420
000137        L = NDIM + 1
000140        DO 7 N = L.9
000141        LABL(I) = AWORD1
000143 7      I = I + 1
000145        LABL(I) = AWORD4
000150        LABL(I+1) = AWORD5
```

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000152          DTP = 0.                                                  TSO 44C
000153          TIME = 0.                                                 TSO 45C
000154          MB = MAXBAN + 1                                           TSO 46C
000156          NE1 = NEN + 1                                             TSO 47C
       C.... INITIALIZE INCREMENTAL FORCE VECTOR                          TSO 48C
000157          DO 6 N = 1.NSICD                                          TSO 49C
000160   6      DF(N) = 0.0                                               TSO 50C
000165          DO 10 N = 1.NUMEL                                         TSO 51C
000166   10     IX(NE1.N) = 0                                             TSO 52C
       C.... PUT INITIAL DATA ON TAPE                                     TSO 53C
000177          ISZH = IBUF                                               TSO 54C
000201          ITRD = ITP13                                              TSO 55C
000202          ITWR = ITP14                                              TSO 56C
000204          REWIND ITRD                                               TSO 57C
000206          IF(IBLK.GT.0) WRITE(ITRD) ((U(I.J).I=1.NDF).J=1.KU)       TSO 58C
000241          DO 11 N = 1.ISZH                                          TSO 59C
000243   11     H(N) = 0.                                                 TSO 60C
000247          NTB= (NH+ISZH-1)/ISZH                                     TSO 61C
000253          WRITE(ITRD) H                                             TSO 62C
000264          IF(NTB.LE.1) GO TO 13                                     TSO 63C
000272          NT = NTB*2                                                TSO 64C
000273          DO 12 N = 1.NT                                            TSO 65C
000275   12     WRITE(ITRD) H                                             TSO 66C
000314   13       REWIND ITRD                                             TSO 67C
000316          IF(IBLK.GT.0) READ(ITRD) DM                               TSO 68C
000331          NEP = NUMNP + NUMNP                                       TSO 69C
000332          C6 = 0.0                                                  TSO 70C
000333          NSTEP = 0                                                 TSO 71C
000334          NST = 0                                                   TSO 72C
000335          NUMPLT=0                                                  TSO 73C
000336          PROP = 0.                                                 TSO 74C
000337          CFLAG = .FALSE.
000340          DO 900 M = 1.NSEQ                                         TSO 75C
000341          READ(ITP5.1000) DT.NTS.INT.NNI.NNF.NEI.NEF.NPROP.NFORC.C0.DM.NT.NL  TSO 76C
000376          IF(NICD. GT.1) C6= DM                                     TSO 77C
000406          IF(NL.NE.0) NST= NL                                       TSO 78C
000410          NTT = IABS(NST)                                           TSO 79C
000412          DFLAG = .FALSE.                                           TSO 80C
000413          IF (NTT. NE. 0 )        DFLAG = .TRUE.                    TSO 81C
000415          NSTEP=NSTEP+NTS                                           TSO 82C
000417          IF(M.EQ.1) NUMPLT=NT                                      TSO 83C
000422          IF(INT.LE.0) INT = 1                                      TSO 84C
000425          WRITE(ITP6.2000) O.HEAD.TIME.IPG DT.NTS.INT.NNI.NNF.NEI.NEF  TSO 85C
000457          IF(DFLAG) WRITE(ITP6.2032) NST                            TSO 86C
000476          IPG = IPG + 1                                             TSO 87C
000500          IF(NICD.EQ.1) GO TO 1                                     TSO 88C
000501          IF(DT.EQ.0.0) GO TO 901                                   TSO 89C
000502          CALL UPDATE(3)                                            TSO 90C
000504   1      IF(NPROP.GT.0) PROP = PROPLD(TIME.NPROP)                  TSO 91C
000517          IF(NFORC.GT.0.AND.NPROP.GT.0) WRITE(ITP6.4031)            TSO 92C
       C.... READ STRESS PLOT INFORMATION                                 TSO 93C
000536          IF(NUMPLT.LE.0.OR.M.GT.1) GO TO 502                       TSO 94C
000547          REWIND 12                                                 TSO 95C
     1          WRITE(ITP6.2005)                                          TSO 96C
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.555            DO 501 N=1.NUMPLT                                              TSO 97C
.00562          READ(ITP5.1006) (NEDATA(N .I).I=1.3)                           TSO 98C
000600   501    WRITE(ITP6.2006) N.(NEDATA(N .I).I=1.3)                        TSO 99C
000632          CALL PLZERO                                                    TSO100C
000633   500    VFLAG = .FALSE.                                                TSO101C
000634          DO 600 NT = 1.NTS                                              TSO102C
000641          PPROP = PROP                                                   TSO103C
000643          PROP = 1.0                                                     TSO104C
000644          IF(DFLAG)CALL NLNORM(4.MDEG.NT.U.DF.DU.F.PPROP.NCT.NTT.IDEST.IBLK)  TSO105C
000667          IF(NPPOP.GT.0) PROP = PROPLD(TIME+DT.0)                        TSO106C
000701          IF(NFORC.GT.0) CALL RESET(-NFORC.NUMNP.NDF.F)                  TSO107C
000713          NCT = 0                                                        TSO108C
000714   14     REWIND ITWR                                                    TSO109C
000716          IF(IBLK.EQ.0) GO TO 15                                         TSO110C
000720          IF(.NOT.VFLAG) GO TO 20                                        TSO111C
000721          DO 21 I = 1.NDEG                                               TSO112C
000723   21     DF(I) = 0.                                                     TSO113C
000730          GO TO 20                                                       TSO114C
000730   15     DO 19 N = 1.NUMNP                                              TSO115C
000732          DO 18 K = 1.NDF                                                TSO116C
000733          J = IDEST(K.N)                                                 TSO117C
000740          IF(J.EQ.0) GO TO 18                                            TSO118C
000741          IF(VFLAG) GO TO 17                                             TSO119C
000743          DO 16 I = 1.MAXBAN                                             TSO120C
000744   16     A(J.I) = 0.0                                                   TSO121C
000756   17     DF(J) = F(K.N)*PROP                                            TSO122C
000766   18     CONTINUE                                                       TSO123C
000771   19     CONTINUE                                                       TSO124C
000773   20     NH = 1                                                         TSO125C
000774          IF(NTB.GT.1) READ(ITRD) H                                      TSO126C
001614          IF(NCT.GT.0) GO TO 44                                          TSO127C
         C....  OUTPUT THE SOLUTION VECTOR FOR THE CURRENT TIME                TSO128C
001017          IF(NNI.EQ.0.OR.((NT-1)/INT)*INT.NE.NT-1) GO TO 40              TSO129C
001032          MCT = 0                                                        TSO130C
001033          DO 30 N = NNI.NNF                                              TSO131C
001034          MCT = MCT - 1                                                  TSO132C
001036          IF(MCT.GT.0) GO TO 31
001040          IF(NICD.EQ.1) WRITE(ITP6.2001) O.HEAD.TIME.IPG.PPROP.M.NT.
               X    (XHED(I).XH.I=1.NDIM).(UHED(I).UH.I=1.NDF)
001121          IF(NICD.NE.1) WRITE(ITP6.2001) O.HEAD.TIME.IPG.PPROP.M.NT.
               X    (XHED(I).XH.I=1.NDIM).(UHED(I).UH.I=1.NDF).(UHED(I).VH.I=1.NDF)
               X  .(UHED(I).AH.I=1.NDF)
001235          IPG = IPG + 1                                                  TSO136C
001237          MCT = 50                                                       TSO137C
001240   31     IF(NICD.EQ.1)WRITE(ITP6.LABL)N.(XYZ(I.N).I=1.NDIM).(U(I.N)
               X    .I=1.NDF)
001311          IF(NICD.NE.1)WRITE(ITP6.LABL)N.(XYZ(I.N).I=1.NDIM).(U(I.N).I=1.NDF
               X ) .(U(I.NUMNP+N).I=1.NDF).(U(I.NEP+N).I=1.NDF)
001420   30     CONTINUE
         C....  UPDATE (1) * * FOR DYNAMIC SOLUTIONS ONLY                      TSO139C
001423   40     IF(NICD.GT.1) CALL UPDATE(1.MDEG.NICD.U.U(1.NUMNP+1).U(1.NEP+1).  TSO140C
               X    DU.F.DF.IDEST.PROP)                                        TSO141C
001457          IF(IBLK.GT.0) WRITE(ITWR) ((U(I.J).I=1.NDF). J=1.KU )          TSO142C
001512   44     MP = 0                                                         TSO143C
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001513          TEMP = 0.                                                      TS01440
001514          DO 400 N = 1,NUMEL                                             TS01450
001515          NPR = .TRUE.                                                   TS01460
001516          DO 43 I = 1,9                                                  TS01470
001520          NPLT(I,1) = 0                                                  TS01480
001521    43    NPLT(I,2) = 0                                                  TS01490
001524          IF(NCT.GT.0) GO TO 46                                          TS01500
001526          IF(N.GE.NEI.AND.N.LE.NEF) NPR = .FALSE.                        TS01510
001540          IF(NUMPLT.LE.0) GO TO 46                                       TS01520
001542          DO 45 I = 1,NUMPLT                                             TS01530
001543          IF(NEDATA(I,1).NE.N) GO TO 45                                  TS01540
001546          J = NEDATA(I,2)                                                TS01550
001547          NPLT(J,1) = I                                                  TS01560
001551          NPLT(J,2) = NEDATA(I,3)                                        TS01570
001553    45    CONTINUE                                                       TS01580
001556    46    MA = MOD(IX(NEL1,N),100)                                       TS01590
001567          IF(MR.LE.0) MRR= IX(NEL1,N)/1000                               TS01600
001600          IF(MR.LE.0) MR = MRR                                           TS01610
001603          DO 60 I = 1,NSTF                                               TS01620
001605          FORCE(I,1) = 0.                                                TS01630
001612          FORCE(I,2)=0.                                                  TS01640
001616          LD(I) = 0                                                      TS01650
001620          IF(MR.NE.MRR.OR.VFLAG) GO TO 60                                TS01660
001625          DO 50 J = 1,NSTF                                               TS01670
001627    50    ESTIF(I,J) = 0.0                                               TS01680
001640    60    CONTINUE                                                       TS01690
001643          L = 0                                                          TS01700
001644          DO 110 I = 1,NEN                                               TS01710
001645          K = IX(I,N)                                                    TS01720
001653          DO 90 J = 1,NDIM                                               TS01730
001654    90    X(J,I) = 0.                                                    TS01740
001661          IF(K.EQ.0)  GO TO 120                                          TS01750
001662          NEL = I                                                        TS01760
001663          DO 100 J = 1,NDIM                                              TS01770
001665   100    X(J,I) = XYZ(J,K)                                              TS01780
001701          DO 110 J = 1,NDF                                               TS01790
001702          L = L + 1                                                      TS01800
001704          DUL(J,I) = DU(J,K)                                             TS01810
001713          UL(J,I) = U(J,K)                                               TS01820
001722          IF(NCT.GT. 0) UL(J,I) =UL(J,I) +DUL(J,I)                       TS01830
001731          IF(NICD.EQ.1) GO TO 110                                        TS01840
001734          UDL(J,I) = U(J,K+NUMNP)                                        TS01850
001744          UDDL(J,I) = U(J,K+NEP)                                         TS01860
001754          IF(NCT.GT.0) UDDL(J,I)= UDDL(J,I) +CS*DUL(J,I)                 TS01870
001765   110    LD(L) = IDEST(J,K)                                             TS01880
002001   120    DM = TYPE(MA)                                                  TS01890
002004          IF(DM.NE.TEMP) MCT = 0                                         TS01900
002007          TEMP = DM                                                      TS01910
002010          CALL TICTOC(TYME,5)                                            TS01920
        C.....  COMPUTE ELEMENT STRESSES AND UPDATE FORCES                     TS01930
002013          CALL ELMLIB(N,MA,NDIM,NDF,NEL,NEL1,NSTF,NSIZV,NVEC,MCT,DM,D,XYZ,  TS01940
               X DM,H,FORCE,ESTIF,U,VECT,5)                                    TS01950
 047            CALL TICTOC(TYME,5)                                            TS01960
        C.....  FORM STIFFNESSIF NEEDED FOR THE NEXT TIME STREP                TS01970
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002052         IF((.NOT.VFLAG.AND.MR.EQ.MPR).OR.(VFLAG.AND.IX(NE1.N).EQ.1))       TS0198C
              XCALL ELMLIB(N.MA.NDIM.NDF.NEL.NEL1.NSTF.NSIZV.NVEC.MCT.IM.D.XYZ.    TS0199C
              X IX.H.FORCE.ESTIF.U.VECT.3)                                         TS0200C
002131         IF(MOD(IX(NEL1.N),1000)/100.GT.0.AND..NOT.VFLAG)                    TS0201C
              X CALL PPTMAT(HEAD.IPG.N.NSTF.ESTIF.FORCE.LD.NSTF.0)                 TS0202C
        C....  MODIFY FOR THE DISPLACEMENT B.C.                                    TS0203C
002165         CALL MODIFY(NDF.NEL.NEL1.NE1.IBLK.NSTF.PROP.IX.ICOD.F.FORCE.ESTIF.  TS0204C
              X    N)                                                              TS0205C
002205         IF(VFLAG) GO TO 300                                                 TS0206C
002212         IF(IBLK.EQ.0) CALL COREIN(A.DF.NEQB.ESTIF.FORCE.LD.NSTF)            TS0207C
002231         IF(IBLK.GT.0) WRITE(7) ESTIF.FORCE.LD                               TS0208C
002263         IF(.NOT.FLAGC.OR.CFLAG) GO TO 400
002266         L = 1
002267         DO 130 I = 1.NEL
002270         K = IX(I.N)
002276         DO 130 J = 1.NDF
002277         DU(J.K) = DU(J.K) + FORCE(L.2)
002311  130    L = L + 1
002317         GO TO 400                                                           TS0209C
002320  300    CONTINUE                                                            TS0210C
        C....  ADD THE FORCE TO THE SOLUTION FOR A RESOLVE                         TS0211C
002320         DO 310 K = 1.NSTF                                                   TS0212C
002322         J = LD(K)                                                           TS0213C
002325         IF(J.GT.0) DF(J) = DF(J) + FORCE(K.1)                               TS0214C
002335  310    CONTINUE                                                            TS0215C
002340  400    CALL TICTOC(TYME.3)                                                 TS0216C
002351         IF(FLAGC.AND..NOT.CFLAG) CALL CONTAC(2.IX.NEL1.NDF.NUMNP.DU)
002364         CFLAG = .TRUE.
002365         IF(NTB.GT.1) WRITE(ITWR) H                                          TS0217C
002405         IF(IBLK.GT.0) WRITE(ITWR) (IX(NE1.N).N=1.NUMEL)                     TS0219C
002436         IF(.NOT.VFLAG) CALL SOLVEQ(NUMNP.NUMEL.NDF.ID1.MB.MAXBAN.9.NSTF.    TS0220C
              1    ISZA.NEQB.IBLK.A.DU.DF.IDEST.FORCE.ESTIF.LD.MAXB.NDEG)          TS0221C
002476         IF(VFLAG) CALL RESVEQ(NUMNP.NDF.MB.MAXBAN.ISZA.NEQB.IBLK.A.DF.DF.   TS0222C
              1    IDEST.IDEST.MAXB.IFLG)                                          TS0223C
002524         CALL TICTOC(TYME.4)                                                 TS0224C
        C....  UPDATE THE SOLUTION                                                 TS0225C
002527         IF(NT.EQ.NTS.AND.M.EQ.NSEQ) GO TO 960                               TS0226C
002542         IF(NCT.GT.0) GO TO 410                                              TS0227C
002544         DTP = DT                                                            TS0228C
002545         I = ITRD                                                            TS0229C
002546         ITRD = ITWR                                                         TS0230C
002547         ITWR = I                                                            TS0231C
002550  410    IF(IBLK.GT.0) BACKSPACE ITRD                                        TS0232C
002554         IF(IBLK.GT.0) READ(ITRD) (IX(NE1.N).N=1.NUMEL)                      TS0233C
002605         REWIND ITRD                                                         TS0234C
002607         IF(IBLK.GT.0) READ(ITRD) ((U(I.J).I=1.NDF).J=1.KU)                  TS0235C
002642         IF(DFLAG) CALL NUNORM(3.NDEG.NT.U.DF.DU.F.PROP.NCT.NTT.IDEST.IBLK)  TS0236C
002666         VFLAG = .TRUE.                                                      TS0237C
002667         IF(NCT.LT.0) VFLAG = .FALSE.                                        TS0238C
002672         NCT = NCT + 1                                                       TS0239C
002674         IF(.NOT.IFLG) GO TO 700
002677         WRITE(6.2000) NCT.UNORM.ENORM.ANORM
               IF(NCT.LT.NTT.AND.(UNORM.GT.UNORM*EPS)) GO TO 14                    TS0240C
          700  IF(DFLAG) DFLAG=.FALSE.
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        C.... UPDATE (2)                                                    TS02450
002733        CALL UPDATE(2,MDEG,NICD,U,U(1,NUMNP+1),U(1,NEP+1),DU,F,DF,IDEST, TS02460
             X   PROP)                                                      TS02470
002751        IF(FLAGC) CALL CONTAC(3,IX,NEL1,NDF,NUMNP,DU,U,U(1,NUMNP+1),
             X          U(1,NEP+1))
003016        MP = MP - 1                                                   TS02480
003020  800   TIME = TIME + DT                                              TS02490
003024  900   CONTINUE                                                      TS02500
003027        IF(NUMPLT.GT.0) CALL PLOTGO(NDIM,NEL,DM,U)                    TS02510
003040        CALL TICTOC (TIME,6)                                          TS02520
003043        WRITE(ITP6,2033) TYME                                         TS02530
003052        RETURN                                                        TS02540
003053  901   WRITE(ITP6,2030)                                              TS02550
003057        IPG = 0                                                       TS02560
003060        RETURN                                                        TS02570
        C.... FORMATS                                                       TS02580
003061  1000  FORMAT(F10.0,8I5,2F10.0,2I5)                                  TS02590
003061  1006  FORMAT(3I5)                                                   TS02600
003061  2000  FORMAT(A1,12A6,E13.5,17X,4HPAGE,I4//5X,23HTIME DEPENDENT SOLUTION TS02610
             X//10X,17HTIME INCREMENT  = 1PE15.4/                           TS02620
             X  10X,17HNUMBER OF STEPS = I5/                                TS02630
             X  10X,17HPRINT INTERVAL  = I5/                                TS02640
             X  10X,14HPRINT NODES    ,I5,3H TO,I5/                         TS02650
             X  10X,14HPRINT ELEMENTS,I5,3H TO,I5)                          TS02660
003061  2001  FORMAT(A1,12A6,E13.5,17X,4HPAGE,I4//5X,12HNODAL VALUES,5X,    TS02670
             X    27HPROPORTIONAL LOAD FACTOR = ,F8.3,5X, 8HSEQUENCE,I4,5X,
             X   9HTIME STEP,I4//12H NODAL POINT,9(1X,2A6),(12X,9(1X,2A6)) )
003061  2002  FORMAT(10X,17HITERATION LIMIT =,I5/1X)                        TS02700
003061  2005  FORMAT(4X,50H DESCRIPTION OF STRESS EVOLUTION PLOTS TO BE MADE.  / TS02710
             X  5X,10H PLOT NO.  5X,8H ELEMENT ,7X, 9H XYZ-CODE,6X,9H SIG-CODE/) TS02720
003061  2006  FORMAT(4(7X,I5,3X))                                           TS02730
003061  2030  FORMAT(5X,53H**FATAL ERROR 33** TIME STEP ZERO FOR DYNAMIC PROBLEM TS02740
             X 1)                                                           TS02750
003061  2033  FORMAT       22X,12HELAPSED TIME //10X,25HINPUT PROPERTIES AND MESH TS02760
             ,F10.3 10X,25HCHECK AND PLOT INPUT DATA,F10.3 10X,14HFORM STIFFNESS TS02770
             ,F21.3 10X,21HSOLUTION OF EQUATIONS,F14.3/10X,15HOUTPUT STRESSES, TS02780
             ,F20.3 10X,27HIMPLICIT ALGORITHM    DISPL.,F9.3 10X,19HTOTAL TIME/IN TS02790
             PLICIT,F16.3 1)                                                TS02800
003061  4001  FORMAT(5X,39H**WARNING  2**  BOTH THE PROPORTIONAL LOADING AND FOR TS02810
             CES ARE BEING RESET ON EACH TIME STEP )                        TS02820
003061  5000  FORMAT 5X,4HCT,I5,  UNORM,E15.5,  DNORM,E15.5,  ANORM,E15.5)   TS02420
003061        END
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14.11.57 OUTPUT
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               SUBROUTINE UPDATE(ISW,NDEG,NICD,U,UD,UDD,DU,F,DF,IDEST,PROP)           UPD   1C
          C***** UPDATE     ****** 12/14/73  ************************************      UPD   2C
000016         DIMENSION U(1),DU(1),F(1),DF(1),IDEST(1),UD(1),UDD(1)                  UPD   3C
000016         COMMON/TAPES/ ITP5,ITP6                                                UPD   4C
000016         COMMON/TIMHIS/ TIME,DT,DTP,NH,ISZH,C0,C1,C2,C3,C4,C5,C6,NCT            UPD   5C
000016         GO TO (100,300,500), ISW                                               UPD   6C
          C.... UPDATE (1) * * PREUPDATE OF ACCELERATIONS FOR DYNAMIC SOLUTIONS.      UPD   7C
000024    100  DO 200 N = 1,NDEG                                                      UPD   8C
000026    200  UDD(N) = C4*UDD(N) - C5*UD(N)                                          UPD   9C
000035         RETURN                                                                 UPD  10C
          C.... UPDATE (2) * * UPDATE THE SOLUTION FOR GENL. DISPL. VEL. ACCL.        UPD  11C
000035    300  DO 400 N = 1,NDFG                                                      UPD  12C
000037         DU(N) = DF(N)                                                          UPD  13C
000043         IF(IDEST(N).EQ.0) DU(N) = F(N)*PROP - U(N)                             UPD  14C
000053         TEMP = DU(N)                                                           UPD  15C
000056         IF(NICD.EQ.1) GO TO 400                                                UPD  16C
000060         P = UDD(N)                                                             UPD  17C
000061         UD(N) = C1*UD(N) + C2*P + C3*TEMP                                      UPD  18C
000067         UDD(N) = P + C6*TEMP                                                   UPD  19C
000073    400  U(N) = U(N) + TEMP                                                     UPD  20C
000100         RETURN                                                                 UPD  21C
          C.... UPDATE(3) * * SET INTEGRATION CONSTANTS                               UPD  22C
000100    500  C6 = C6 + 0.5                                                          UPD  23C
000102         IF(C0.EQ.0.0) C0 = 0.25                                                UPD  24C
000104         WRITE(ITP6,2002) C0,C6                                                 UPD  25C
000114         C5 = 1./C0/DT                                                          UPD  26C
000116         C4 = 1. - 0.5/C0                                                       UPD  27C
000121         C3 = C6*C5                                                             UPD  28C
000123         C2 = (1. - C6/C0/2.)*DT/C4                                             UPD  29C
000127         C1 = 1. - C6/C0 + C2*C5                                                UPD  30C
000134         C6 = C5/DT                                                             UPD  31C
000137         RETURN                                                                 UPD  32C
000137    2002 FORMAT(10X,30HNEWMARK INTEGRATION PARAMETERS/                          UPD  33C
              C10X17HBETA VALUE     = F6.3/                                           UPD  34C
              C10X17HGAMMA VALUE    = F6.3)                                           UPD  35C
000137         END
```